MODIFICATION DESIGN WITH MINIMAL ROTARY INERTIA FOR THE PITCH CURVE WITH CONCAVE CUSPS OF N-LNG

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ABSTRACT: To improve machining quality of teeth profile and transmission performance of N-lobed noncircular gear (N-LNG), the calculus of variations is employed to establish a modification model of design defect for the pitch curve with concave cusps that takes rotary inertia into consideration. A general minimal rotary inertia modification algorithm for pitch curve with concave cusps can be developed based on the model. The pitch curve without concave cusps for any type of N-LNG can be obtained by applying the proposed method. Outer contact and inner contact pitch curves conjugated with the modified pitch curve can also be obtained by using the principle of gearing. Several typical examples are implemented in computer, and simulation results demonstrate that the research should be helpful in the design and manufacture of N-LNG.

KEYWORDS: noncircular gear, pitch curve design, concave cusp defect, mathematical model, minimal rotary inertia modification

1 INTRODUCTION

Noncircular gear pair has been used to implement variable transmission and function generation with applications in pump, printing, packaging, automotive steering, aeronautical ring dampers, knee motion assist mechanism, power drive mechanism and etc (Litvin, 2004, 2008), (Dooner, 2001), (Stefano, 2012), (Terada, 2012), (Mundo, 2006). However, because of the complex of design and manufacture for noncircular gears with discretionary pitch curves, its application is not as well as circular gears widely. One of important reasons is most research on noncircular gears focused on N-lobed or high-order elliptical gears (Bair, 2002), (Tsay, 2005), (Tong, 1998). Figliolini G. and co-workers proposed a general generation method of N-lobed elliptical gears from a basic ellipse, and the transmission mathematical models among cutter, driving and driven elliptical gears can be established in their studies (Figliolini, 2000, 2003, 2005). Moreover, the synthesis of high-order elliptical gears and its hobbing machining method can be obtained (Lin, 2013), (Zhang, 2014), (Figliolini, 2016).

More recently, some novel pitch curves of noncircular gears can be studied so as to expand its application. For examples, Hector, Salvador and et al designed an approach for generating pitch curves of N-lobed noncircular gears based on Bézier and B-spline nonparametric curves (Hector, 2007). Yao presented a pitch curve design method, which using plane regular N-curved polygon and spiral of Archimedes as the pitch curve for N-lobed noncircular gear (Yao, 2013). The Archimedes spiral, quadratic curve and Pascal curve are applied in pitch surface design of noncircular bevel gear (Lv, 2016). Noncircular gears with steepest rotation pitch curves can be synthesized, and the research is applied to modify the pitch curve with discontinuity points (Zhang, 2016a, 2016b).

Because of the complex of manufacture for pitch curve with cusps, the pitch curve should be reshaped to improve the machining process. Therefore, the minimal rotary inertia modification model of pitch curve with concave cusps for N-LNG can be established by the kinematics and calculus of variations in this paper due to the large rotary inertia will increase mechanical load and reduce response of the transmission system. Besides, we develop a modification algorithm for the design of conjugate pitch curves without concave cusps for N-LNGs. Numerical results are shown to illustrate the proposed method should be helpful in the design and manufacture of noncircular gears. They should also be helpful in the low load and high response of this type of gearing.
2 MODIFICATION MODEL OF THE PITCH CURVE WITH CONCAVE CUSPS

As shown in Fig.1, points A, B and O are non-collinear points in the plane, using straight line connect the points O and A, and points O and B, respectively. Let the X-axis of fixed coordinate system \( I(O-XY) \) coincide with the line OA. While using discretionary plane curve \( r_1(\eta) \) connects points A and B, then \( \angle AOB \) is center angle of curve \( r_1(\eta) \), polar angle \( \eta \) is measured counterclockwise from the positive direction X-axis. In order to ensure that the closed of the noncircular gear pitch curve, curve \( r_1(\eta) \) should satisfy the following conditions:

\[
\begin{align*}
\angle AOB &= 2\pi / N \\
r_1(0) &= r_1(2\pi / N)
\end{align*}
\]

where \( N \) is an integer number.

Therefore, discretionary closed pitch curve \( r(\eta) \) of N-LNG can be formed by rotating the plane curve \( r_1(\eta) \) around the center \( O \).

As shown in Fig.2, points \( c_1, c_2, ..., c_N \) may be concave cusps (i.e. continuous but non-differentiable points) of the pitch curve \( r(\eta) \) for N-LNG. The design and machining of N-LNG have great difficulties due to the presence of these concave cusps. Modification of the pitch curve with concave cusps is necessary and important according to processing requirements of N-LNG and positions of these concave cusps have a certain rotated period \( 2\pi/N \). For convenience, therefore, we just need to study the modification model of one concave cusp. Assuming the modification pitch curve corresponding concave cusp \( c_k \) \((k=1,2,...,N)\) is \( r_M(\eta) \), which used to replace two tiny section pitch curves (curves \( r_1, r_2 \)), \( a_k \) and \( b_k \) are points of tangency that the modification pitch curve \( r_M(\eta) \) makes with pitch curve \( r_k(\eta) \) and \( r_{N-1}(\eta) \), respectively. Without additional explanation, the following discussion relating to subscript \( k \) \((k=1,2,...,N)\), if \( k=N \), then \( (k+1)\equiv1 \), and if \( k=1 \), then \( (k-1)\equiv N \). Assuming the polar angles of points \( a_k \) and \( b_k \) are \( \eta_{ak} \) and \( \eta_{bk} \) respectively. So the infinitesimal arc \( dS \) of modification pitch curve \( r_M(\eta) \) can be expressed as:

\[
dS = \sqrt{r_M^2(\eta) + r_M'2(\eta) d\eta}
\]

where subscript ‘MK’ in curve \( r_M(\eta) \) is refer to that curve \( r_M(\eta) \) is the modification pitch curve \( r_M(\eta) \) for pitch curve \( r_k(\eta) \) at the concave cusp \( c_k \).

N-lobe closed plane curve \( r(\eta) \) can be obtained by rotating the curve \( r_1(\eta) \) around the center \( O \), it can be expressed as:

\[
r(\eta) = M \begin{cases} 
  r_1(\eta), & \eta \in [0, \frac{2\pi}{N}] \\
  r_1(\eta - \frac{2(k-1)\pi}{N}), & \eta \in \left[\frac{2(k-1)\pi}{N}, \frac{2k\pi}{N}\right] 
\end{cases}
\]

where \( k=1,2,...,N \).
Substituting Eq.(3) into Eq.(4), the infinitesimal rotary inertia differential $dJ$ can be obtained:

$$dJ = \frac{m [r_{Mk}^3(\eta) + r_{Mk}^2(\eta)]}{\int_{\eta_{a}}^{\eta_{b}} \sqrt{r_{Mk}^2(\eta) + r_{Mk}^2(\eta)} d\eta}$$

(5)

The integral to both sides of Eq.(5) at the same time, one obtains:

$$J[r_{Mk}(\eta)] = \int_{\eta_{a}}^{\eta_{b}} m [r_{Mk}^3(\eta) + r_{Mk}^2(\eta)] \sqrt{r_{Mk}^2(\eta) + r_{Mk}^2(\eta)} d\eta$$

(6)

Referring to Eq.(6), we know that the rotary inertia $J$ may be different for different modification pitch curves $r_{Mk}(\eta)$, the minimum value $J_{min}$ of rotary inertia $J$ can be expressed as:

$$J_{min} = \min_{r_{Mk}(\eta) \in C^1[\eta_{ak}, \eta_{bk}]} \int_{\eta_{a}}^{\eta_{b}} F d\eta$$

(7)

where $r_{Mk}(\eta) \in C^1[\eta_{ak}, \eta_{bk}]$ refers to $r_{Mk}(\eta)$ have 2-order successive derived function at $\eta \in \{\eta_{ak}, \eta_{bk}\}$.

According to the calculus of variations (Lao, 2015) and generic function $F$ of Eq. (7) doesn’t contain parameter $\eta$, the first integral of Euler-Lagrange Equation can be expressed as:

$$F - r_{Mk}^3(\eta) F_{\eta_{a,c}} = 0$$

(8)

where $h$ is an integral constant.

Substituting $F$ of Eq.(7) into Eq.(8), we can obtain:

$$\frac{r_{Mk}^4(\eta)}{\sqrt{r_{Mk}^2(\eta) + r_{Mk}^2(\eta)}} = h_{1}$$

(9)

where $h_{1} = \frac{m}{\int_{\eta_{a}}^{\eta_{b}} \sqrt{r_{Mk}^2(\eta) + r_{Mk}^2(\eta)} d\eta}$.

In order to avoid the occurrence of new concave cusps after the changes, the modified pitch curve $r_{Mk}(\eta)$ with minimal rotary inertia should meet the following differential conditions:

$$\begin{cases}
   r_{Mk}^3(\eta_{a}) = r_{1}^3(\eta_{a}) \\
   r_{Mk}^3(\eta_{b}) = r_{1}^3(\eta_{b})
\end{cases}$$

(10)

Along with Eq.(9), modification model with minimal rotary inertia for the pitch curve with concave cusps of N-LNG can be obtained:

$$\begin{align}
   r_{Mk}^4(\eta) & = h_{1} \\
   \sqrt{r_{Mk}^2(\eta) + r_{Mk}^2(\eta)} & = h_{2}
\end{align}$$

(11)

where $s.t.$ is an abbreviation of “subject to”, it means that contained constraint conditions must be satisfied by the equation.

3 ANALYSIS AND SOLUTION OF THE MODIFICATION MODEL

The solution of Eq.(11) is called the modification pitch curve $r_{Mk}(\eta)$ with minimal rotary inertia corresponding concave cusp $c_{k}$. Referring to Eq.(11), assuming:

$$r_{Mk}^3(\eta) = r_{Mk}^3(\eta) \tan \mu$$

(12)

Substituting Eq.(12) into Eq.(11), we can obtain:

$$dr_{Mk}(\eta) = \frac{r_{Mk}^3(\eta) \tan \mu}{3} d\mu = \frac{r_{Mk}^3(\eta)}{3} d\mu$$

(13)

$$d\eta = \frac{dr_{Mk}(\eta)}{r_{Mk}^3(\eta)} = \frac{d\mu}{3}$$

(14)

The integral to both sides of Eq.(15) at the same time, one obtains:

$$3\eta = \mu + h_{2}$$

(15)

where $h_{2}$ is an integral constant.

Along with Eq.(13), the modification pitch curve $r_{Mk}(\eta)$ with minimal rotary inertia can be expressed as:

$$r_{Mk}(\eta) = \frac{h_{1}}{\cos(3\eta - h_{1})}, \eta \in \{\eta_{ak}, \eta_{bk}\}$$

(16)

where $h_{1}$, $h_{2}$ are undetermined constants, polar angles $\eta_{ak}$ and $\eta_{bk}$ must satisfy the following constraints due to the value of pitch curve $r_{Mk}(\eta)$ is constant greater than zero:

$$\eta \in [\eta_{ak}, \eta_{bk}] \subset \left(\frac{\pi}{3}, \frac{\pi}{3} + \frac{2\pi}{3}, \frac{\pi}{3} + \frac{2\pi}{3}, \frac{\pi}{3}, \frac{\pi}{3} + \frac{2\pi}{3}\right), h_{1} < 0$$

(17)

$$\eta \in [\eta_{ak}, \eta_{bk}] \subset \left(\frac{\pi}{3}, \frac{\pi}{3} + \frac{2\pi}{3}, \frac{\pi}{3} + \frac{2\pi}{3}, \frac{\pi}{3}, \frac{\pi}{3} + \frac{2\pi}{3}\right), h_{1} > 0$$

(18)

where $i = \ldots, -2, -1, 0, 1, 2, \ldots$

According to Eq.(12) and Eq.(16), the first-order derivative of modification pitch curve $r_{Mk}(\eta)$ can be obtained:
Along with Eqs.(17)~(19), we can obtain:

\[
\begin{align*}
\tan(3\eta - h_2) < 0, \eta &\in \left( \frac{h_2}{3}, \frac{2\pi}{3} + \frac{2i\pi}{3} + \frac{h_2}{3} \right), h_2 < 0 \quad (21) \\
\tan(3\eta - h_2) > 0, \eta &\in \left( \frac{h_2}{3}, \frac{2\pi}{3} + \frac{2i\pi}{3} + \frac{h_2}{3} \right), h_2 > 0 \quad (22)
\end{align*}
\]

\[
\begin{align*}
\tan(3\eta - h_2) < 0, \eta &\in \left( \frac{h_2}{3}, \frac{2\pi}{3} + \frac{2i\pi}{3} + \frac{h_2}{3} \right), h_2 < 0 \quad (23) \\
\tan(3\eta - h_2) > 0, \eta &\in \left( \frac{h_2}{3}, \frac{2\pi}{3} + \frac{2i\pi}{3} + \frac{h_2}{3} \right), h_2 > 0 \quad (24)
\end{align*}
\]

According to boundary conditions \( r_{Ma}(\eta_{a_k}) = r_{Ma}(\eta_{b_k}) \) and differential conditions \( r_{Ma}'(\eta_{a_k}) = r_{Ma}'(\eta_{b_k}) \), undetermined constants \( h_1, h_2 \) and polar angles \( \eta_{a_k}, \eta_{b_k} \) can be obtained by the following conditions:

\[
\begin{align*}
\tan(3\eta - h_2) &\in \left[ \frac{h_1}{\cos(3\eta - h_2)}, \frac{h_1}{\cos(3\eta - h_2)} \right] \\
\tan(3\eta - h_2) &\in \left[ \frac{h_1}{\cos(3\eta - h_2)}, \frac{h_1}{\cos(3\eta - h_2)} \right]
\end{align*}
\]

Referring to Fig.2, we known that the first-order derivatives \( r_{Ma}'(\eta_{a_k}) \) and \( r_{Ma}'(\eta_{b_k}) \) around the concave cusp \( c_k \) should satisfy the following conditions:

\[
\begin{align*}
r_{Ma}'(\eta_{a_k}) < 0 \\
r_{Ma}'(\eta_{b_k}) > 0
\end{align*}
\]

According to Eqs.(17)~(26), the polar equation of the modification pitch curve \( r_{Ma}(\eta) \) with minimal rotary inertia can be expressed as:

\[
r_{Ma}(\eta) = \sqrt{\frac{h_1}{\cos(3\eta - h_2)}}, \eta \in [\eta_{a_k}, \eta_{b_k}]
\]

Along with Eq.(2), the sketch of modified pitch curve \( r_{Ma}(\eta) \) is shown in Fig.3, and the polar equation \( r_{Ma}(\eta) \) can be expressed as:

\[
\begin{align*}
r_{Ma}(\eta) &= \sqrt{\frac{h_1}{\cos(3\eta - h_2)}}, \eta \in [\eta_{a_k}, \eta_{b_k}] \\
r_{Ma}(\eta) &= \sqrt{\frac{h_1}{\cos(3\eta - h_2)}}, \eta \in [\eta_{a_k}, \eta_{b_k}]
\end{align*}
\]
Furthermore, the mathematical model of pitch curves conjugated with the modified pitch curve \( r_M(\eta) \) which contain outer contact and inner contact can be expressed, respectively:

\[
\begin{align*}
\eta_a &= \int_0^a \frac{r_M(\eta)}{a_0 - r_M(\eta)} \, d\eta \\
\eta_i &= \int_0^a \frac{r_M(\eta)}{a_1 + r_M(\eta)} \, d\eta
\end{align*}
\]

where \( r_{OM}(\eta_0) \), \( r_{OM}(\eta_1) \) and \( a_0 \), \( a_1 \) are the pitch curves and center distances of outer contact and inner contact noncircular gears conjugated with the modified pitch curve \( r_M(\eta) \), respectively.

In order to ensure the closure of pitch curves \( r_{OM}(\eta_0) \) and \( r_{OM}(\eta_1) \) conjugated with the modified pitch curve \( r_M(\eta) \), the center distances \( a_0 \) and \( a_1 \) can be adjusted by the following conditions:

\[
\begin{align*}
\frac{2\pi}{N_0} &= \int_0^{2\pi} \frac{r_M(\eta)}{a_0 - r_M(\eta)} \, d\eta \\
\frac{2\pi}{N_1} &= \int_0^{2\pi} \frac{r_M(\eta)}{a_1 + r_M(\eta)} \, d\eta
\end{align*}
\]

where \( N_0 \) and \( N_1 \) are the numbers of lobes of pitch curves \( r_{OM}(\eta_0) \) and \( r_{OM}(\eta_1) \).

4 MODIFICATION ALGORITHM

Modification algorithm with minimal rotary inertia characteristic for the pitch curve with concave cusps of N-LNG was produced using the foregoing formulations. Referring to the algorithm flow chart of Fig. 4, the pitch curve \( r(\eta) \) with concave cusps for N-LNG is the known condition, concave cusp \( c_k \) can be arbitrarily chosen by designers, arbitrarily. Polar angles \( \eta_{a_k}, \eta_{b_k} \) and undetermined constants \( h_1, h_2 \) can be solved by resorting to Eq.(25), Eq.(27) or Eq.(25), Eq.(28). The modified pitch curve \( r_M(\eta) \) can be obtained through Eqs.(27)–(30), its conjugate pitch curves \( r_{OM}(\eta_0) \) and \( r_{OM}(\eta_1) \) can also be obtained by Eqs.(31)–(34).

5 MODIFICATION EXAMPLES

The proposed modification method with minimal rotary inertia characteristic was implemented in computer to run several typical examples, as shown in Figs. 5–8. For convenience, the value of subscript \( k \) for selected concave cusp \( c_k \) is 1, without additional explanation, the following examples relating to subscript \( k \) of selected concave cusp \( c_k \) whose value is 1.

Fig. 5 is the 3-lobed noncircular gear pitch curve with concave cusps, its polar equation \( r(\eta) \) can be expressed as:

\[
\begin{align*}
r(\eta) &= r_i(\eta) + \sqrt{3} \cos(\eta), \eta \in [0, \frac{2\pi}{3}]
\end{align*}
\]

Along with Eq.(25) and Eq.(27) or Eq.(25) and Eq.(28), undetermined constants \( h_1, h_2 \) and polar angles \( \eta_{a_k}, \eta_{b_k} \) of modification pitch curve \( r_M(\eta) \) for the pitch curve \( r(\eta) \) with concave cusps in Fig. 5 can be obtained, respectively:
Substituting Eq.(36) or Eq.(37) into Eq.(27) that takes the periodic characteristic of trigonometric function into consideration, the polar equation of the modification pitch curve \( r_{M1}(\eta) \) with minimal rotary inertia can be obtained:

\[
r_{M1}(\eta) = \sqrt{\frac{6 \sqrt{3}}{\cos(3\eta)}}, \eta \in \left[ \frac{7\pi}{12}, \frac{3\pi}{4} \right] (38)
\]

Then the modified pitch curves \( r_M(\eta) \) for the pitch curve \( r(\eta) \) with concave cusps in Fig. 5 can be obtained:

\[
\begin{align*}
\eta & = \frac{7\pi}{12} \\
\eta & = \frac{3\pi}{4} \\
\eta & = \frac{17\pi}{12} \\
\eta & = \frac{23\pi}{12} \\
\eta & = \frac{25\pi}{12}
\end{align*}
\]

Fig. 6(b) is the modified pitch curves \( r_{M}(\eta) \) with the minimal rotary inertia for the pitch curve \( r(\eta) \) with concave cusps in Fig. 5, its conjugate pitch curves equations and graphs which contain outer contact and inner contact are described in Eqs. (40)–(41) and Figs. 6(b)–(c).

Referring to the modification process of 3-lobed noncircular gear pitch curve with concave cusps, the modified pitch curves \( r_M(\eta) \) with the minimal rotary inertia for the pitch curve \( r(\eta) \) with concave cusps in Fig. 7 is depicted in Fig. 8(a), its conjugate pitch curves which contain outer contact and inner contact are described in Figs. 8(b)–(c).
Fig. 7 4-lobe noncircular gear pitch curve with concave cusps

Corresponding modification parameters of the pitch curve with concave cusps in Fig. 7 are listed in Table 1, and the polar equations of modified pitch curve \( r_M(\eta) \) can be expressed as:

\[
\begin{align*}
    r_s(\eta) &= \sin(\eta) + \cos(\eta), \quad \eta \in \left[ \frac{-\pi}{16}, \frac{7\pi}{16} \right] \\
    r_{s1}(\eta) &= \sqrt{2\sqrt{2} \cos\left(\frac{3\pi}{16}\right)}, \quad \eta \in \left[ \frac{7\pi}{16}, \frac{9\pi}{16} \right] \\
    r_i(\eta) &= r_s(\eta - \frac{\pi}{2}), \quad \eta \in \left[ \frac{9\pi}{16}, \frac{15\pi}{16} \right] \\
    r_{i1}(\eta) &= r_s(\eta - \frac{\pi}{2}), \quad \eta \in \left[ \frac{15\pi}{16}, \frac{17\pi}{16} \right] \\
    r_{si}(\eta) &= r_s(\eta - \frac{3\pi}{2}), \quad \eta \in \left[ \frac{17\pi}{16}, \frac{21\pi}{16} \right] \\
    r_{si1}(\eta) &= r_s(\eta - \frac{3\pi}{2}), \quad \eta \in \left[ \frac{21\pi}{16}, \frac{33\pi}{16} \right] \\
\end{align*}
\]

Referring to Figs. 5-8, we know that the modification accuracy is higher with the decrease of the \( \Delta \eta \) (\( \Delta \eta = \eta_{b1} - \eta_{a1} \)). Therefore, designer can choose appropriate polar angles \( \eta_{a1} \) and \( \eta_{b1} \) to ensure the requirement of engineering accuracy.

Table 1. Modification parameters of the pitch curve \( r(\eta) \) with concave cusps in Fig. 7

<table>
<thead>
<tr>
<th>Polar angles ( \eta_{a1}, \eta_{b1} )</th>
<th>Integral constants ( h_1, h_2 )</th>
<th>Modified pitch curve ( r_M(\eta) )</th>
<th>Conjugate pitch curves</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta_{a1} = \frac{7\pi}{16} )</td>
<td>( h_1 = -2\sqrt{2} \cos\left(\frac{3\pi}{16}\right) )</td>
<td>( r_{M1}(\eta) = \sqrt{2\sqrt{2} \cos\left(\frac{3\pi}{16}\right)} )</td>
<td>Outer contact ( \eta = \frac{7\pi}{16} )</td>
</tr>
<tr>
<td>( \eta_{b1} = \frac{9\pi}{16} )</td>
<td>( h_2 = \frac{3\pi}{2} )</td>
<td></td>
<td>Inner contact ( \eta = \frac{9\pi}{16} )</td>
</tr>
</tbody>
</table>

\( r_{M1}(\eta) = \sqrt{2\sqrt{2} \cos\left(\frac{3\pi}{16}\right)} \) (Referring to Fig. 8(b))

\[ \eta = \int_{\eta_{a1}}^{\eta_{b1}} \frac{r_{M1}(\eta)}{2.5827 - r_{M1}(\eta)} \, d\eta \]

(Referring to Fig. 8(c))
6 REFERENCES


7 NOTATION

The following symbols are used in this paper:

- $a_i, b_i$: points of tangency
- $a_1, a_0$: center distances of the pitch curves
- $r_{DM}(\eta)$ and $r_{OM}(\eta_0)$: concave cusps
- $dm$: infinitesimal mass
- $dJ$: infinitesimal rotary inertia
- $dS$: infinitesimal arc length
- $h_1, h_2$: undetermined constants
- $m$: mass of noncircular gear pitch curve
- $r(\eta)$: pitch curve equation of N-LNG
- $r_{OM}(\eta)$: modified pitch curve equation
- $r_{OM}(\eta)$: modification pitch curve equation
- $r_{OM}(\eta_1)$, $r_{OM}(\eta_0)$: pitch curve equations of outer contact and inner contact noncircular gears conjugated with the modified pitch curve $r_{OM}(\eta)$
- $N$: number of lobes
- $N_0, N_1$: numbers of lobes of pitch curves $r_{OM}(\eta_0)$ and $r_{OM}(\eta)$
- $J$: rotary inertia
- $J_{min}$: minimum value of rotary inertia $J$
$S =$ arc length of the modification pitch curve $r_{Mk}(\eta)$;
$\eta_{ak}$, $\eta_{bk}$ = Corresponding polar angles of points $a$ and $b$;
$\lambda$ = linear density;
$\Gamma(O-XY)$ = fixed coordinate system.