INTEGRATED ASSIGNMENT AND FACILITY LOCATION APPROACH BASED ON BLACK HOLE OPTIMIZATION

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ABSTRACT: The interconnected supply chain solutions make logistic processes so complex that only integrated design applications makes it possible to offer suitable solutions. After a systematic literature review, it was found that heuristic algorithms are important tools for design of interconnected complex systems since a wide range of models determines a NP-hard optimization problem with a wide range of objective functions and constraints. Within the frame of this article authors present a black hole algorithm based metaheuristic approach which focuses on the integrated design of single-tier supply chain of manufacturing companies using multi-sources. The sensitivity analysis of the developed optimization algorithm is tested with benchmark functions. Numerical results with different datasets demonstrate the usability of the integrated design algorithm.

KEYWORDS: assignment, convergence, facility location, heuristic optimization, logistics, supply chain

1 INTRODUCTION

Nowadays a company cannot prosper alone; it needs a high level of cooperation with its suppliers, customers, service providers and rivals too. This creates not only virtual but physical channels too, which leads to a high complexity network in manufacturing, storing, supplying as well as other parts of the companies life. If a company coordinates these physical channels properly, it can get a big advantage. Unfortunately, a bigger network means a much bigger problem, because everything connects to everything in different routes. Without a proper system, we can create chain events. To solve a problem in a network environment normal linear tools are usually not as effective as some heuristic approaches.

The main contribution of this work include: (1) an integrated supply chain model that combines assignment and facility location problems in the case of a single tier supply; (2) a black hole optimization based algorithm to solve the integrated model; (3) a test of the modified black hole algorithm with different data sets from the point of view convergence characteristics; (4) computational results of integrated design.

Since our study embraces several related research streams, namely supply chain design and heuristic algorithms, we provide a brief review on each stream before to elaborate the model, algorithm and solution. There are different types of integrated design methods, depending on the combined design tasks. Mixed integer linear programming is used for facility location and capacity optimization for commodity chemicals production via woody biomass fast pyrolysis (Zhang et al., 2014). Linear and nonlinear optimization models have been widely used to solve integrated supply chain design problems, where inventory costs and distribution costs needs to take into consideration (Shen, 2007).

In cellular manufacturing systems the cell formation is the core decision problem, but the material supply of them also influences the operability, especially in dynamic environment, where the system parameters are stochastic variables. Depending on managerial perspectives, the integrated design can include the procurement, manufacturing and distribution processes and the solution method is a multi-choice goal programming with fuzzy parameters (Paydar & Saidi-Mehrabad, 2015).

The reliability of supply chain structure is another important aspect of integrated design. Exact and metaheuristic methods can be used for the analysis of integrated supply chain design reliability, where managerial perspectives can also be taken into consideration. Subsequently, not only structure reliability but also efficiency has a great impact on the operation of supply chain (Ivanov et al., 2016). The performance indicators of technology and logistics have a great impact on each other, therefore it is important to integrate different design tasks, like scheduling (Gubán & Gubán, 2012), assignment, inventory management, routing facility location.
The design problem of supply chain network, which consists of one external supplier, a set of potential distribution centers, and a set of retailers, each of which is faced with uncertain demands for multiple commodities can be solved with nonlinear integer programming (Wu & Zhang, 2014).

In the case of supply chain solutions with high complexity level integrated solution methodology can solve NP-hard problems, where the combination of different heuristic algorithms indicates a good convergence characteristics and repeatability (Kota & Jármai, 2017). The performance of the combination of the Taguchi technique with Artificial Immune System and Genetic Algorithm is good from the point of view convergence characteristics to optimize an integrated supply chain design problem with multiple shipping (Tiwari et al., 2010). The application of black hole algorithm is shown in different research works focusing on complexity problems of interconnectivity (Bányai et al., 2017; Bányai et al., 2015).

Another typical integrated design problem is the integrated supply chain design problem that determines the locations of retailers and the assignments of customers to retailers to minimize the expected costs of location, transportation, and inventory (Qi et al., 2010). A Lagrangian relaxation based solution algorithm is proposed to solve the integrated design problem of incorporating inventory and routing (Max Shen & Qi, 2007).

Mixed integer convex programming is used for simultaneous optimization of location, allocation, capacity, inventory, and routing decisions in a stochastic supply chain system (Ahmadi Javid & Azad, 2010).

The design aspect of complex supply chain solutions can include not only logistics related objective functions, but also cost functions can be taken into account using a multi-criteria optimization (Elmaraghy & Majety, 2008). The main types of cost functions can be highlighted as follows: linear cost function, quadratic cost function and cubic cost function. Linear cost function can be used integrating customer responsiveness and distribution cost in designing a two-echelon supply chain network with considering capacity level (Azad & Ameli, 2008).

2 MATERIALS AND METHODS

As a strategic supplier of a company the deliveries have to be calibrated precisely and sent in time especially if we use the principle of Just in Time. These strategic suppliers have their own system, usually fully connected to the costumer company. But only a few suppliers can call themselves strategically important, the rest of them are low tier suppliers, who usually have to arrange usually themselves the storage and deliveries of goods themselves. Today society and economy standards like grouping a networking gives us advantages. We have to create a reliable supply network for lower tier suppliers too, to increase our stability and market position (see in Figure 1).

![Model of warehousing in lower tier supply network](image)

Fig. 1 Model of warehousing in lower tier supply network

For a reliable source of parts and goods, we can establish regional warehouses near the suppliers and arrange collection routes.

This way the cost of transport will decrease, because the suppliers do not have to organize the delivery individually and the vehicle capacity utilization can increase. On the other hand, building and maintaining warehouses are expensive. In the rest of the article, we will show how we can determine the number of warehouses and their locations based on the costs and parameters of suppliers.

We can calculate the total cost of delivery and operating warehouses based on the following equation:

\[
TC_{TP} = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ID} \cdot 2 \cdot L_{\text{LTS},\text{RWH}_j} \cdot \frac{TF}{F_{\text{LTS}_i}} + \\
\sum_{j=1}^{m} C_{OD} \cdot 2 \cdot L_{\text{RWH}_j,\text{AP}} \cdot \frac{TF}{F_{\text{RWH}_j}} + \\
\cdot C_{\text{Oper}} \cdot TF + j \cdot C_{\text{build}} \rightarrow \text{min.}
\]  

Equation (1)

The equation can be divided into four parts. In the first part we calculate the cost of collecting the goods from suppliers and bringing them into the assigned warehouse, based on the delivery frequency and distance from the warehouse. The second part is the cost of organized deliveries from the warehouses to the company; it uses the same principle as the first part. The third part is the
operation cost of the warehouse. We use an average operating cost, but it usually contains several kinds of sub-cost, like energy and heating, salary, administration, vehicle use, security and insurance and of course the storage cost of goods. This cost increases with the number of warehouses. The last part is the building or purchasing cost of the warehouse. It needs to be paid only once, but it is a big investment. We can use this equation for different time frames, for example when the investment returns.

There is also an inventory and capacity management task to take into consideration. The easiest way to assign a supplier to a warehouse is to choose the closest one. But if there are too much suppliers with high storage demands in a region, there are two simple way to solve the problem.

- The warehouse uses a more dynamic storage and handling for faster throughput.
- Share the goods with a nearby warehouse.

The second option however raises the Knapsack problem. Which goods should go to a further location? For this, we have to examine several parameters, like size, weight, frequency of order, special treatment or can it be grouped with other goods. From these parameters we can calculate the “usefulness” of the goods and arrange them optimally:

\[ SCAP_{RWH_i} \geq \sum SDEM_{LTS_i}(\text{arranged}) \]  \hspace{1cm} (2)
\[ \sum U_{LR} \rightarrow \max. \]  \hspace{1cm} (3)

We use these equations and objective function in the application for solving a localization problem. The optimization algorithm, which produced the solutions of the case below, was the black hole algorithm. It is a reliable and easily manageable method for problems, which needs heuristics to solve it (see Figure 2).

Fig. 2 Impact of photon sphere and event horizon on moving particles and photons (Bányai et al, 2019)

The black hole optimization algorithm is a population-based nature inspired implicit method. Black hole algorithm was first mentioned in 2008, as random black hole particle swarm optimization tool used for benchmarking functions. Since then the algorithm got several improvements many of them also nature based like the Hawking radiation. Figure 3 shows the Pseudo code of the algorithm.

The black hole algorithm can be controlled by the following equations:
\[ P_{i}^{t+1} = P_{i}^{t} + \text{rand}(0 \ldots 1) \times (P_{BH}^{t} - P_{i}^{t}) \]  
\[ R^{t} = \frac{F_{BH}^{t}}{\sum_{i=1}^{n} P_{i}^{t}} \text{ if } P_{i}^{t} \text{ is a parameter of a star and PBH is the same parameter but for the black hole.} \]

The first equation describes the movement of a parameter between the \( t \) and \( t+1 \) iteration. \( P_{i} \) is a parameter of a star and PBH is the same parameter but for the black hole.

**Input:** number of stars, objective function, constraints, sign restrictions, termination criteria  
**Output:** optimal solution

(1) generate solutions randomly in the dimensional search space  
(2) for each stars, evaluate the objective function  

While (termination criteria satisfy) do  
(3) select the best star that has the best value to become a hole  
(4) change the position of the black hole  
(5) move the stars towards the black holes while constraints are taken into consideration  
(6) if star is inside the Event horizon absorb the star and generate a new one in the search space  
(7) for each stars, evaluate the objective function  
End of while

**Fig. 3** Pseudo code of the black hole algorithm

The event horizon is calculated by the second equation, where we divide the value of objective function of the black hole with the summarized value of all stars. There are other improved methods to calculate the movement or the event horizon that involves gravitational force, acceleration or mass consumption where the stars can pull each other or the black hole can move too.

In the case study we use 20 suppliers in a 50km radius of the Assembly plant which has a coordinate (50, 50) as Figure 4 shows.

We simulate scenarios where we try to localize between 1 and 10 warehouses to collect the goods from the suppliers. In this case we do not take into account the inventory and limited storage of the warehouse. We give every supplier a delivery frequency from 1 month up to 7 times a week. The total time frame set to 3 years, because in the industry that is the average return time of an investment. The parameters for the case study are the followings:

- Total cost of building or buying a warehouse: 200000 USD  
- Number of weekly organized deliveries to the assembly plant: 14

**Fig. 4** Map of the suppliers; the optimal location of three warehouses and the assembly plant

After this methodological part we would like to analyze and validate the described mathematical model and the proposed metaheuristics based on black hole algorithm. Our methodology for the validation of the mathematical model of facility location problem is that we calculate the optimal facility locations within the frame of various scenarios using different data sets including input data for the physical layout planning problem and the parameters of the black hole based metaheuristic algorithm.

After running the scenarios with different number of warehouses, we noticed an oddity. No matter how many warehouses we try to localize; there is always one at the assembly plant. Those warehouses only move a little bit further from the assembly plant when we give highly unrealistic parameters to the application (see Table 1).

Figure 5. shows the four different costs in million USD that we mentioned earlier. The cost of operation and warehouse building are constantly rising with the number of warehouses.

On the other hand the cost of collection routes from suppliers to warehouses decreases exponentially, while the cost of organized transport increases slowly. This gives us the solution in this situation that we should invest in 3 warehouses, if we want to minimize the cost of a 3 year running.
We used an application created by us, to tune our system and easily predict certain things. For this reason, we need to create models, that capable of describing the complexity and large amounts of data for time. As further research direction we would like to enhance our earlier black hole algorithm (Veres et al., 2017) and verify the increased efficiency and convergence performance. Another research direction is the integrated approach of facility location problems, where various aspects, like human resource management (Manzoor et al., 2017) and verify the increased efficiency and convergence performance. As further research direction we would like to enhance our earlier black hole algorithm (Veres et al., 2017) and verify the increased efficiency and convergence performance.

### Table 1. Cost and coordinates of different number of warehouses

<table>
<thead>
<tr>
<th>No. of warehouses</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost in timeframe (M USD)</td>
<td>1,4484</td>
<td>1,4130</td>
<td>1,3014</td>
<td>1,4916</td>
<td>1,7209</td>
<td>1,9161</td>
<td>2,1778</td>
<td>2,3965</td>
<td>2,6190</td>
<td>2,8001</td>
</tr>
<tr>
<td>WH1</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
</tr>
<tr>
<td>WH2</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
</tr>
<tr>
<td>WH3</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
</tr>
<tr>
<td>WH4</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
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<td>Y Coord</td>
<td>X Coord</td>
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</tr>
<tr>
<td>WH5</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
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<tr>
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<td>X Coord</td>
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<tr>
<td>WH7</td>
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<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
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</tr>
<tr>
<td>WH8</td>
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<td>Y Coord</td>
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<td>Y Coord</td>
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</tr>
<tr>
<td>WH9</td>
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<td>Y Coord</td>
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<td>X Coord</td>
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</tr>
<tr>
<td>WH10</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
<td>X Coord</td>
<td>Y Coord</td>
</tr>
</tbody>
</table>

**Fig. 5** Cost of different numbers of warehouses

### 3 CONCLUDING REMARKS

To acquire solution in a complex supply network we need to create models, that capable of describe its complexity and large amounts of data for to fine tune our system and easily predict certain things. Our model described a solution for the lower tier supplier network enhanced by cooperation in warehousing. We used an application created by us, in a case study to simulate and calculate the right location of certain number of warehouses taken into account the different cost that occurs in a timeframe. We do this by the aid of a fast and reliable heuristic method, the black hole algorithm.

As further research direction we would like to enhance our earlier black hole algorithm (Veres et al., 2017) and verify the increased efficiency and convergence performance. Another research direction is the integrated approach of facility location problems, where various aspects, like human resource management (Manzoor et al., 2017), routing, technological conditions and real time supply chain scheduling.

### 4 ACKNOWLEDGEMENTS

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### 5 REFERENCES


Bányai Á., Bányai T., Illés B. (2017) Optimization of Consignment-Store-Based Supply Chain with


Table 2. Symbols used in this paper

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation of the symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTSₐ</td>
<td>iʰ low tier supplier</td>
</tr>
<tr>
<td>TCₜf</td>
<td>Total cost of the examined timeframe</td>
</tr>
<tr>
<td>C_ID</td>
<td>Cost of individual delivery to a warehouse</td>
</tr>
<tr>
<td>C_OD</td>
<td>Cost of organized delivery from warehouse to the assembly plant</td>
</tr>
<tr>
<td>C_Oper</td>
<td>Cost of operation of a warehouse in a timeframe</td>
</tr>
<tr>
<td>C_build</td>
<td>Cost of building a warehouse</td>
</tr>
<tr>
<td>LₗTSᵢⱼRWHⱼ</td>
<td>Length of route between the supplier and assigned warehouse</td>
</tr>
<tr>
<td>LᵩRWH_APP</td>
<td>Length of route between a warehouse and the assembly plant</td>
</tr>
<tr>
<td>TF</td>
<td>Examined time frame</td>
</tr>
<tr>
<td>F_LTₐᵢ</td>
<td>Frequency of delivery from the iʰ supplier</td>
</tr>
<tr>
<td>F_RWHⱼ</td>
<td>Frequency of delivery from the jʰ warehouse</td>
</tr>
<tr>
<td>SCAPᵩᵣRWHⱼ</td>
<td>Storage capacity of the arranged warehouse</td>
</tr>
<tr>
<td>SDEMₗTSᵢ(arranged)</td>
<td>Storage demand of arranged suppliers</td>
</tr>
<tr>
<td>U</td>
<td>Usefulness of the goods</td>
</tr>
<tr>
<td>LR</td>
<td>Length of route</td>
</tr>
<tr>
<td>n</td>
<td>Number of low tier suppliers; i=1…n</td>
</tr>
<tr>
<td>m</td>
<td>Number of regional warehouses; j=1…m</td>
</tr>
</tbody>
</table>