THE MULTIVARIABLE NEW INFORMATION GREY MODEL WITH RECIPROCAL ACCUMULATED GENERATING OPERATION AND ITS APPLICANT TO DATA PROCESS OF MANUFACTURING TECHNOLOGY

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ABSTRACT: Data process is the basis of modern manufacturing engineering. Based on the Grey system and accumulated generating operation of reciprocal number, a multivariable Grey new information model MGRM(1,n) which was taken the m-th component as the initialization to the tribological behaviors analysis of hydronium plating films was put forward. The Grey multi-variable new information MGRM(1,n) model is generalized for GRM(1,1) model under n variables. The simultaneous solution the MGRM (1,n)model’s parameters can reflect the restricted and affected multi-variables relation with one another. The proposed model has high precision and easy to use. Examples of the tribological behaviors analysis with MGRM (1,3) model were given. The results from the examples show the model is simple and practical, may instruct the design and use of the films in engineering.

Keyword: Accumulated Generating Operation of Reciprocal Number, Grey MGRM (1, n) Model, Film, Tribological Behaviors, Data Processing, Advanced Manufacturing Technology

1. INTRODUCTION

Experimental data processing in advanced manufacturing technology can be regarded as a system, while the factors in the system are interdependent and interacting with each other (Melian, C., Pasca, I., Lobontiu, M., 2013). The interaction between various factors is uncertain and gray. Thus, it cannot reflect the mutual influence, coordinated development and constraints among multiple variables only by utilizing single time series data. The friction coefficient and wear rate of the film coating are comprehensive indexes of its tribological property, which are greatly influenced by external factors such as load and velocity. At different loads and velocities, the friction coefficient and wear rate of the film coating are also different. However, in the existing tribological theory, the mapping relationship between the external factors (such as load and velocity) and the tribological properties of the film (friction coefficient and wear rate) has not been established. Based on gray system with few data, the gray system theory can process the known data series by data transformation to establish a unique differential equation model. By fully exploiting the explicit and implicit information in less data, it can find the ordered data from disordered data, and then infer the future development law (Sifeng Liu, Bo Zeng, Jiefang Liu, et al., 2015; Liu Qingping, Xu Xiaoci, He Youcheng, 2003). When the GM (1, 1) model is used to predict the tribological property of film coating, it only considers the influence of single factor with the prediction method, resulting in the inaccuracy (Liu Sifeng, Zeng Bo, LIU Jie-fang, et al., 2014). A gray GM (1, n) model is proposed in Literature(Liu Sifeng, Dang Yaoguo, Fang Zhigeng, et al., 2014; Li Yan,Cheng Hongbo, Xin Jianbo, et al.,2017). On the one hand, the establishment of the model should confirm the main factor. In the project, the confirmation of main factor in many isocentric
factors is quite difficult. On the other hand, due to the fitting or prediction inaccuracy, the model can be used for qualitative analysis instead of fitting and prediction. Based on the engineering requirements of multivariable and high-precision GM (1, n) model, the optimized GM (1, n) model is established by optimizing the gray derivative background value and the parameter identification method (Li Yan, Cheng Hongbo, Xin Jianbo, et al., 2017). The optimized GM (1, n) model expands the application of the GM (1, n) model. Moreover, it solves the problems that cannot be solved by the existing GM (1, n) and GM (1,1) models. However, the calculation method of this model is more complicated. Gray prediction MGM (1, n) model with multiple related variables is proposed in Literature (Shu Fuhua, 2016). MGM model is the generalization of the GM (1,1) model in the case of n variables. The parameters of MGM model can reflect the mutual influence and mutual restraint relationships between multiple variables. Using fractional order to describe the object with fractional order property can better reveal their nature and behavior. The grey model FGM (1,1) with fractional order accumulation was proposed and the weapons maintenance fee was predicted using FGM (1,1) (Fang Shili, Wu Lifeng, Yu Liang, et al., 2013). The discrete grey model DFGM (1,1) with fractional order accumulation was proposed (Lifeng Wu, Sifeng Liu, Ligen Yao, et al., 2014). FGM (1,1) and DFGM (1,1) was summarized and the applicable range of FGM (1,1) was researched (Wei Meng, Bo Zeng, 2015). The optimal solution of FGM (1,1) was studied (Yue Shen, Ping Qin, 2014). FGM (1,1) with single variable and fractional order was extended to the multivariate grey model FMGM(1, n) with fractional order (Qiyuan LIU, Dejie YU, 2016).

For the original sequence \( x^{(1)} \) with monotonous decreasing, the reciprocal accumulation generation is proposed to establish the gray GRM (1, 1) model (HE Xia, LIU Wei-feng, 2011). By further improving the gray GRM (1, 1) model, the improved gray GRM (1, 1) model is established based on reciprocal accumulation generation (HE Jun, Lin Yin-ping, 2015). The non-equidistance Grey Model GRM(1,1) is established based on reciprocal accumulation generation (Bohong Zheng, LanXiang Luo, 2015). In terms of both the reciprocal accumulation generation and inverse accumulation generation, the established models make the generated sequence \( x^{(1)} \) monotonically decrease. Then \( x^{(1)} \) is fitted by a monotonically decreasing curve to derive its model value \( \hat{x}^{(1)} \). After reducing \( \hat{x}^{(1)} \), the predicted value \( x^{(0)} \) of \( \hat{x}^{(0)} \) is finally obtained. Through these modeling methods, it does not generate the unreasonable error as the traditional accumulation and subtraction modeling method, thus improving the modeling accuracy.

The work firstly proposed the multivariable new information MGRM (1, n) model based on the reciprocal accumulation generation. In the modeling process, the modeling method in terms of reciprocal accumulation generation was combined with the gray prediction MGM(1,n) model containing multiple related variables. According to the grey new information theory, the m-th component of the original data sequence was regarded as the initial condition of the gray differential equation. Then the established MGRM (1, n) model was applied to analyze the tribological properties of film coating. The model with simple calculation and high accuracy provides the basis for the design and practical application of film coating.

2. MULTIVARIABLE NEW INFORMATION GRAY MGRM (1, N) MODEL BASED ON THE RECIPROCAL ACCUMULATION GENERATION

Definition 1. Set the sequence
Let $x_i^{(1)}(k)(i = 1,2,\cdots,n)$ be the corresponding one-order accumulation generation sequence of $x_i^{(0)}(k)(i = 1,2,\cdots,n)$, i.e
\[
x_i^{(1)}(k) = \sum_{j=1}^{k} x_i^{(0)}(j), (j = 1,2,\cdots,m)
\]

(1)

MGRM (1, n) model is the first order differential equations with n variables
\[
\begin{align*}
\frac{dx_1^{(1)}}{dt} &= a_{11}x_1^{(1)} + a_{12}x_2^{(1)} + \cdots + a_{1n}x_n^{(1)} + b_1 \\
\frac{dx_2^{(1)}}{dt} &= a_{21}x_1^{(1)} + a_{22}x_2^{(1)} + \cdots + a_{2n}x_n^{(1)} + b_2 \\
&\vdots \\
\frac{dx_n^{(1)}}{dt} &= a_{n1}x_1^{(1)} + a_{n2}x_2^{(1)} + \cdots + a_{nn}x_n^{(1)} + b_n
\end{align*}
\]

(2)

Let $X^{(0)}(k) = (x_1^{(0)}(k), x_2^{(0)}(k), \cdots, x_n^{(0)}(k))^T$

\[
X^{(1)}(k) = (x_1^{(1)}(k), x_2^{(1)}(k), \cdots, x_n^{(1)}(k))^T
\]

\[
A = \begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1} & a_{n2} & \cdots & a_{nn}
\end{bmatrix}, \quad B = \begin{bmatrix}
b_1 \\
b_2 \\
\vdots \\
b_n
\end{bmatrix}
\]

then Equation (2) can be denoted as
\[
\frac{dX^{(1)}}{dt} = AX^{(1)} + B
\]

(3)

According to the new information priority principle of gray system theory, taking the first component $x_i^{(1)}(1)$ of the sequence
\[
x_i^{(1)}(j)(j = 1,2,\cdots,m)
\]

as the initial condition of the gray differential equation will result in insufficient utilization of new information. Instead, using the m-th $x_i^{(1)}(m)$ component can make full use of the latest information. Then the continuous time response of Equation (3) is
\[
X^{(1)}(t) = e^{At}X^{(1)}(m) + A^{-1}(e^{At} - I)B
\]

(4)

Where, $e^{At} = I + \sum_{k=1}^{\infty} \frac{A^k t^k}{k!}$, I is unit matrix

To distinguish A and B, equation (2) is discretely processed to obtain
\[
x_i^{(0)}(k) = \sum_{j=1}^{n} \frac{a_i}{2} (x_i^{(1)}(k) + x_i^{(1)}(k-1)) + b_i
\]

(5)

\[i = 1,2,\cdots,n; k = 2,3,\cdots,m\]

Let $a_i = (a_{i1}, a_{i2}, \cdots, a_{in}, b_i)^T (i = 1,2,\cdots,n)$, then the predicted value $\hat{a}_i$ of $a_i$ can be obtained by the least squares method.
\[
\hat{a}_i = [\hat{a}_{i1}, \hat{a}_{i2}, \cdots, \hat{a}_{in}, \hat{b}_i]^T = (Z^TZ)^{-1}Z^TY_i
\]

(6)

\[i = 1,2,\cdots,n\]

Where,
\[
Z = \begin{bmatrix}
\frac{1}{2}(x_i^{(0)}(2) + x_i^{(0)}(1)) & \frac{1}{2}(x_i^{(0)}(2) + x_i^{(0)}(0)) & \cdots & \frac{1}{2}(x_i^{(0)}(2) + x_i^{(0)}(0)) & 1 \\
\frac{1}{2}(x_i^{(0)}(3) + x_i^{(0)}(2)) & \frac{1}{2}(x_i^{(0)}(3) + x_i^{(0)}(2)) & \cdots & \frac{1}{2}(x_i^{(0)}(3) + x_i^{(0)}(3)) & 1 \\
\frac{1}{2}(x_i^{(0)}(m) + x_i^{(0)}(m-1)) & \frac{1}{2}(x_i^{(0)}(m) + x_i^{(0)}(m-1)) & \cdots & \frac{1}{2}(x_i^{(0)}(m) + x_i^{(0)}(m-1)) & 1 \\
\end{bmatrix}
\]

\[Y_i = [x_i^{(0)}(2), x_i^{(0)}(3), \cdots, x_i^{(0)}(m)]^T\]

Then, the discriminant values of A and B

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can be obtained.

\[
\hat{A} = \begin{bmatrix}
\hat{a}_{11} & \hat{a}_{12} & \cdots & \hat{a}_{1n} \\
\hat{a}_{21} & \hat{a}_{22} & \cdots & \hat{a}_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\hat{a}_{n1} & \hat{a}_{n2} & \cdots & \hat{a}_{nn}
\end{bmatrix}, \quad \hat{B} = \begin{bmatrix}
\hat{b}_1 \\
\hat{b}_2 \\
\vdots \\
\hat{b}_n
\end{bmatrix}
\]

(7)

The calculated value of the new information MGRM (1, n) model is

\[
\hat{X}^{(1)}(j) = e^{\hat{\lambda}(j-m)}X^{(1)}(m) + \hat{A}^{-1}(e^{\hat{\lambda}(j-m)} - 1)\hat{B}
\]

(8)

\[j = 1,2,\cdots,m\]

The above equation takes the m-th component of the original data sequence as the initial condition of the gray differential equation. After reduction, the fitting value of the original data is as follows.

\[
\lim_{\Delta t \to 0} \frac{\hat{X}^{(0)}(j) - \hat{X}^{(0)}(j - 1)}{\Delta t} = \hat{X}^{(0)}(j), \ j = 2,3,\cdots,m
\]

According to Definition 1, the model value \(\hat{X}^{(0)}(j)(j = 1,2,\cdots,m)\) of the original sequence can be obtained.

The absolute error of the i-th variable is defined as

\[q_i(k) = \hat{X}^{(0)}(i)(k) - X^{(0)}(i)(k)\]

(10)

The relative error of the i-th variable (%) is defined as

\[e_i(k) = \frac{\hat{X}^{(0)}(i)(k) - X^{(0)}(i)(k)}{X^{(0)}(i)(k)} \times 100\]

(11)

The average value of relative error of the i-th variable is defined as

\[f = \frac{1}{m} \sum_{k=1}^{m} |e_i(k)|\]

(12)

3. MODEL TESTS

After establishment, the model should be tested to determine whether the model is suitable. Three methods are commonly used to test the MGRM (1, n) model (Sifeng Liu, Bo Zeng, Jiefang Liu, et al., 2015; Liu Qingping, Xu Xiaoci, He Youcheng, 2003; Liu Sifeng, Dang Yaoguo, Fang Zhigeng, et al., 2014): residual test, correlation test and posterior variance test (omitted).

For the multivariable new information gray MGRM (1, n) model based on reciprocal accumulation generation, its modeling process is recorded as NMGRM with Matlab2017b programming. When the original data is input, various required data and model test results can be output.

4. TRIBOLOGICAL PROPERTY ANALYSIS OF FILM COATING

4.1 Tribological property analysis of TiN film coating

Table 1 shows the test data of TiN film coating under the load of 600 N and the relative sliding velocities of 0.314, 0.417, 0.628, 0.942 and 1.046 m/s, respectively.

<table>
<thead>
<tr>
<th>No.</th>
<th>k</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sliding velocity (m/s)</td>
<td>0.314</td>
<td>0.471</td>
<td>0.628</td>
<td>0.942</td>
<td>1.046</td>
</tr>
<tr>
<td></td>
<td>Friction coefficient ( \mu )</td>
<td>0.251</td>
<td>0.258</td>
<td>0.265</td>
<td>0.273</td>
<td>0.288</td>
</tr>
<tr>
<td></td>
<td>Wear rate ( \omega ) (( \times 10^{-5} ) mg/m)</td>
<td>7.5</td>
<td>8.0</td>
<td>8.5</td>
<td>9.5</td>
<td>11.0</td>
</tr>
</tbody>
</table>

According to the method of this work, the new information MGRM (1, 3) model about...
sliding velocity, friction coefficient and wear rate is established. The parameters of the model are

\[
A = \begin{bmatrix}
1.0897 & 1.8110 & -78.1540 \\
-0.1790 & -0.4128 & 14.9112 \\
0.0120 & -0.0052 & -0.0792 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
2.0761 \\
4.1606 \\
0.1206 \\
\end{bmatrix}
\]

The model values of friction coefficient are 0.25368, 0.25875, 0.26605, 0.27327 and 0.28718, respectively. The relative errors (%) of friction coefficient are 1.0692, 0.29147, 0.39445, 0.099146 and -0.28639, respectively. The average values of relative error of friction coefficient and the model are 0.42814 and 4.1279%, respectively. Thus, the model test is “good”.

4.2 Tribological property analysis of CrN film coating

Table 2 shows the test data of CrN film coating under the load of 600 N and the relative sliding velocities of 0.314, 0.417, 0.628, 0.942 and 1.046 m/s, respectively.

<table>
<thead>
<tr>
<th>No.</th>
<th>Sliding velocity (m/s)</th>
<th>Friction coefficient (\mu)</th>
<th>Wear rate (\omega)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.31</td>
<td>0.32</td>
<td>9.5 (\times 10^{-5}) mg/m</td>
</tr>
<tr>
<td>2</td>
<td>0.47</td>
<td>0.33</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>0.62</td>
<td>0.33</td>
<td>10.5</td>
</tr>
<tr>
<td>4</td>
<td>0.94</td>
<td>0.35</td>
<td>12.5</td>
</tr>
<tr>
<td>5</td>
<td>1.04</td>
<td>0.37</td>
<td>17</td>
</tr>
</tbody>
</table>

According to the method of this work, the new information MGRM (1, 3) model about sliding velocity, friction coefficient and wear rate is established. The parameters of the model are

\[
A = \begin{bmatrix}
0.6393 & 0.7295 & -39.9127 \\
-0.0635 & -0.2248 & 7.3534 \\
0.0207 & -0.0175 & 0.0946 \\
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
2.2452 \\
3.1846 \\
0.0784 \\
\end{bmatrix}
\]

The model values of friction coefficient are 0.32644, 0.33252, 0.34067, 0.3501 and 0.37167, respectively. The relative errors (%) of friction coefficient are 1.0656, 0.45952, 0.49208, 0.027234 and -0.3573, respectively. The average values of relative error of friction coefficient and the model are 0.48035 and 4.5964%, respectively. Thus, the model test is “good”.

Based on the model test results, this model has the adaptability and validity. Thus, it is very necessary to establish this model.

5. CONCLUSIONS

(1) In the work, the m-th component of the original data sequence was used as the initial condition of the gray differential equation. Based on the reciprocal accumulation generation, a multivariable new information gray MGRM (1, n) model was proposed for the tribological property analysis of ion plating film coating. The parameters of MGRM model reflected the mutual influence and mutual restraint relationships between multiple variables. Then, it provided a general multivariable gray model and method for experimental data processing in advanced manufacturing technology.

(2) The model is of high precision, as well as
easy to use. The calculation example shows the correctness and validity of the model. Thus, the establishment of the model has important practical and theoretical significance. It is worth popularizing in the data processing in modern manufacturing technology and experimental monitoring of tribological property analysis of film coating.

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