CONTACT RATIO CALCULATION OF INVOLUTE ARC GEAR DEVICE

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ABSTRACT: Following the basic principles of gear meshing and the computing theories of gear contact ratio, this paper deducts the contact ratio formula of involute arc gear device (IAGD), and analyzes the effect of the ratio. The proposed formula reveals that the IAGD contact ratio is positively correlated with the circular arc curvature and negatively with pressure angle. The research results shed new light on the theory and application of the IAGD design.

KEY WORDS: involute arc gear device (IAGD), contact ratio.

1 INTRODUCTION

As one of the most popular options for gear mechanism, the cylindrical gear device mainly consists of spur gears, helical gears and herringbone gears. To overcome the structural defects of these gears, the arc gear has been developed (Koga, 1975) to achieve the transmission of ordinary cylindrical gear with an improved structure. The new type of gear has attracted the attention of numerous theorists (Tseng, 2004; Dai, 2006; Kou, 2002; Song, 2006; Ma, 2005; Faydor and Litvin, 1994).

Defined as the ratio of the arc of action to the pitch circle, the contact ratio is a structural indicator of gear pair that reflects the continuity, smoothness and uniformity of the transmission load of the gear pair, and illustrates the load distribution between the meshing teeth of the pair. The ratio has a direct bearing on various parameters, such as the noise control of gear transmission, the bending strength of tooth root, the contact strength of tooth surface, the adhesion of tooth surface, etc. (Faydor and Litvin, 1994)

Following the basic principles of gear meshing and the computing theories of gear contact ratio, this paper deducts the contact ratio formula of involute arc gear device (IAGD), and analyzes the effect of the ratio.

2 GENERATION OF IAGD TOOTH SURFACE

As shown in Figure 1, chord CD of circular arc CGD on generating plane S is parallel to busbar NN' of the base cylinder. The radius and center of the arc are \( R_t \) and \( Q_t \), respectively.

As surface \( S \) rotates along the base cylinder, the \( C_0G_0D_0 \) becomes the curve of the circular arc CGD on the cylindrical surface, turning the trajectory of the point on CGD into the tooth surface of the IAGD (Jiang, 2014).

![Figure 1. Generation of the tooth surface of the IAGD](image)

Assuming that the axis of the base cylinder is axis \( z \), the middle radial cross section is coordinate plane \( xoy \), and midpoint \( G_0 \) of circular arc \( C_0G_0D_0 \) is on axis \( y \), the coordinate system \( S(x, y, z) \) can be established on the frame of reference, and the equation of tooth surface in the coordinate system can be acquired as below (Jiang, 2014):

\[
\begin{align*}
    x &= r_t \sin \theta_t - \phi \cos \theta_t \\
    y &= r_t \cos \theta_t + \phi \sin \theta_t \\
    z &= h
\end{align*}
\] (1)
where \( r_b \) is the radius of the base cylinder; \( \theta \) is the turning angle of the generating plane \( S \); \( \phi \) is the opening angle of random point \( M \) on \( CGD \) when \( S \) is rotated.

3 IAGD CONTACT RATIO

3.1 Endface contact ratio of the IAGD

If the IAGD is radially sectioned into infinitely narrow pinions, then each pinion can be approximated as an involute spur gear. The contact ratio of the pinion end face, which is in the shape of involute tooth profile, can be calculated in the same way as that of involute spur gear. Here, the contact ratio is denoted as \( \varepsilon \). Figure 2 shows the tooth profile \( \beta-\beta \) and \( \gamma-\gamma \) of the teeth \( O_1 \) and \( O_2 \) in three different positions.

![Figure 2. Random radial crosssection of involute gear meshing](image)

Hereto, it is possible to examine the rotating angle of the teeth pair of two tooth profiles in one meshing cycle from the beginning to the end of contact. Suppose the pinion angle and the gear angle are \( \angle B_1O_1B_2 \) and \( \angle B_1O_2B_2 \), respectively, the tangential contact of two adjacent tooth profiles is a continuous process under the following conditions

\[
\angle B_1O_1B_2 \geq \frac{2\pi}{z_1} \quad \angle B_1O_2B_2 \geq \frac{2\pi}{z_2}
\]  

(2)

where \( z_1 \) and \( z_2 \) are the teeth number of gears \( O_1 \) and \( O_2 \), respectively.

Thus, the contact ratio can be expressed as:

\[
\varepsilon_a = \frac{l}{P_b} = \frac{l}{p_c \cos \alpha}
\]  

(3)

where \( l = B_1B_2 \) is the length of the working part of the meshing line, that is, the contact point displacement along the meshing line in the gear cycle; \( P_b \) is the measured distance between the two adjacent tooth profiles in the common line direction; \( P_c \) is the distance between the two teeth on the pitch circle; \( \alpha \) is the pressure angle of the gear.

The following equation holds in accordance with Figure 2:

\[
KB_2 + B_1L = KL + l
\]  

(4)

Therefore,

\[
l = r_{hi} \tan \alpha_{hi} + r_{i2} \tan \alpha_{i2} - (r_{hi} + r_{i2}) \tan \alpha
\]  

(5)

where \( r_{hi} \) and \( r_{i2} \) are the base radiuses of the gear pair, respectively.

3.2 Axial direction contact ratio

For the involute spur gear, the mating process develops along the pitch circle (Figure 3(a)). The gear teeth are meshed across the entire tooth width both during the engagement at \( A_1A_2 \) and the disengagement at \( A_2A_2 \). Hence, the contact ratio of the gear equals \( l/P_b \), where \( l \) is the engaging length and \( P_b \) is the end face distance of the normal tooth.

![Figure 3. Contact ratio calculation of the IAGD](image)
According to in Figure 3 (b), the actual meshing length of the IAGD is $\Delta l$ longer than that of the involute spur gear. Therefore, the IAGD has a greater contact ratio than the involute spur gear.

The gear circumferential angle subtended by $\Delta l$ equals the circumferential angle corresponding to the arc teeth in Figure 1, i.e. the circumferential angle $\theta_b$, subtended by the distance of point $G$ to chord $CD$.

$$\theta_b = (R_t - \sqrt{R_t^2 - (B/2)^2})/r_b$$

(6)

Owing to the dependence on tooth width $B$ and circular arc tooth radius $R_t$, the contact ratio corresponding to $\Delta l$ is called the axial contact ratio $e_{\beta b}$ which is expressed as:

$$e_{\beta b} = \frac{\Delta l}{p_b} = \frac{(R_t - \sqrt{R_t^2 - (B/2)^2})\cos\alpha}{p_b}$$

(7)

where $\alpha$ is the standard pressure angle on the end face of the gear; $m_e$ is the axial modulus; $B$ is the tooth width.

### 3.3 IAGD contact ratio

The IAGD contact ratio $e_r$ is the sum of the end face contact ratio and the axial contact ratio:

$$e_r = e_{\alpha r} + e_{\beta r}$$

(8)

The expression of $e_r$ is obtained by substituting formulas (3) and (7) into formula (8):

$$e_r = \frac{P l}{\pi \cos\alpha} + \frac{(R_t - \sqrt{R_t^2 - (B/2)^2})}{\pi m_e}$$

(9)

### 4 INFLUENCING FACTORS OF CONTACT RATIO

It can be seen from formula (9) that arc radius $R_t$ and pressure angle $\alpha$ are directly correlated with the IAGD contact ratio. The two parameters both affect the intuitive geometry of the gear, and determine the shape of the gear tooth profile. Thus, it is very meaningful to explore the relationship between $R_t$, $\alpha$ and the contact ratio. Substituting formula (5) into formula (9), we can acquire the following formula:

$$e_r = \frac{\tan a_{n_1} + r_{b_2} \tan a_{n_2} - (r_{b_1} + r_{b_2}) \tan a}{p_c \cos\alpha}\frac{(R_t - \sqrt{R_t^2 - (B/2)^2})}{\pi m_e}$$

(10)

#### 4.1 Effect of pressure angle on the contact ratio

According to formula (10), the derivative of $a$ is obtained as:

$$\frac{\partial e_r}{\partial \alpha} = \frac{-r_{b_1} \tan a_{n_1} + r_{b_2} \tan a_{n_2} - (r_{b_1} + r_{b_2}) \tan a}{p_c \cos^2\alpha}$$

(11)

As shown in Figure 2,

$$r_{b_1} \tan a_{n_1} + r_{b_2} \tan a_{n_2} - (r_{b_1} + r_{b_2}) \tan a \geq 0$$

and $0 \leq \alpha \leq \pi/2$. Thus, it is concluded that $\frac{\partial e_r}{\partial \alpha} \leq 0$, indicating that the IAGD contact ratio decreases as the pressure angle $\alpha$ increases. In other words, a low pressure angle is favorable for the transmission of the gear device. However, the pressure angle should not be too small, for the chance of rooting cutting grows with the decline in the angle.

#### 4.2 Effect of arc curvature on the contact ratio

Similarly, the derivative of arc radius $R_t$ is acquired as:

$$\frac{\partial e_r}{\partial R_t} = \frac{\sqrt{R_t^2 - (B/2)^2} - R_t}{\pi m_e \sqrt{R_t^2 - (B/2)^2}}$$

(12)

Since $B/2 \leq R_t$, it is concluded that $\partial e_r/\partial R_t \leq 0$. Obviously, the contact ratio grows with the radius $R_t$ of the circular arc.

5 CONCLUDING REMARKS

In this part will be emphasize the contributions of the paper and the future applications in the research field.

(1) According to the principles of gear meshing, the contact ratio formula of the IAGD was deduced, laying the theoretical basis for the design of pressure angle $\alpha$ and arc radius $R_t$ of the involute gear.
(2) The IAGD contact ratio is positively correlated with the circular arc curvature and negatively with pressure angle.

(3) During the parameter design of the IAGD, the proper selection of pressure angle and arc curvature could increase the contact ratio, and thereby ensure the continuous and smooth operation of the gear.

6 REFERENCES