THE IMPORTANCE OF MINKOWSKI SUM IN STAMPING OPERATION DESIGN

Mircea ANCĂU[1]

ABSTRACT: The present paper refers to the practical utility of Minkowski sum in the process of optimal nesting. There are presented methods to draw Minkowski sum for convex, as well as concavely polygonal profiles. Also it is analyzed a method to find an optimal arrangement of profiles on the strip of material, and the calculation of minimum strip width and strip pitch. There are also analyzed the advantages and limitations of these methods found in literature and a very simple method to increase the material utilization coefficient is proposed, especially for "L" types of polygonal profiles. Finally, some numerical examples for the proposed variant of optimization are presented.

KEY WORDS: Optimal nesting, Minkowski sum, polygonal profile.

1 INTRODUCTION

In the stamping tools design process, the most important decision to take is based on the part orientation on the material strip. This orientation decides the efficiency of the material consumption. A stamping tool becomes profitable in a case of high batch size only. A good example for this is the case of automotive industry. If the batch size is high, and the tool design is not optimal, a small percent of material loss at a single manufactured piece can produce considerable loss at the level of the entire production. Because the material cost represents roughly 75% from the whole production cost of a stamped part (Nye, 2000), a stamping tool improper designed will produce material loss along its entire life. Only in some very particular cases, such objective can be achieved by a manually design process. So, the optimization of the material consumption can be achieved by a proper orientation of a polygonal profile on the material strip. Anyway, the optimal orientation is not so simple, while the minimum strip width and pitch are changing along with polygonal profile rotation. The material consumption is a parameter of major importance and is defined as a division of the blank area (the polygonal area of the part) to the material strip area. The choice of polygonal profile orientation on the strip so that the pitch becomes minim, does not necessarily mean an optimal material consumption.

[1] Universitatea Tehnică din Cluj-Napoca, Facultatea de Construcţii de Maşini, Departamentul de Ingineria Fabricaţiei, B-dul Muncii 103-105, Cluj-Napoca, România

E-mail: mircea.ancau@tcm.utcluj.ro.

2 THE MINKOWSKI SUM DRAWING

To avoid the overlapping of two adjacent profiles, the space-obstacle concept from robotics is used. The space-obstacle is represented as a polygon drawn by Minkowski sum. For two polygons A and B, the Minkowski sum is:

$$A \oplus B = \{a + b | a \in A, b \in B\}$$

where a and b are points on poligons A and B, while a+b is a sum vector. Practically, a reference point of one polygon, follow the contour of the other polygon. The Minkowski sum polygon represents the minimum area in which these two polygons touch each other without overlapping. By use of Minkowski sum, it is demostrated (Joshi, 1994) that if the strip width is bigger than profile width, than optimal position of the profile on the material strip coresponds to that with minimum pitch between two consecutive profiles. While the Minkowski sum polygon is only a tool in the process of optimal orientation of the profile to stamp on the material strip, its drawing might become very difficult. At Minkowski sum drawing there are two distinct cases, corresponding to convex and concave polygons.

2.1 Convex polygons

Let's consider the case of two convex polygonal profiles A and B as in figure 1. Let's consider the case of two convex polygonal profiles A and B as in figure 1. We draw the polygon –B, as symmetric to the origin of the Cartesian system xOy.

The a_i and b_i segments belong to the polygons A and –B, and are sorted in ascendant order of the angle made by each segment with the horizontal axis (see fig.2). The reading order of these segments is counterclockwise.

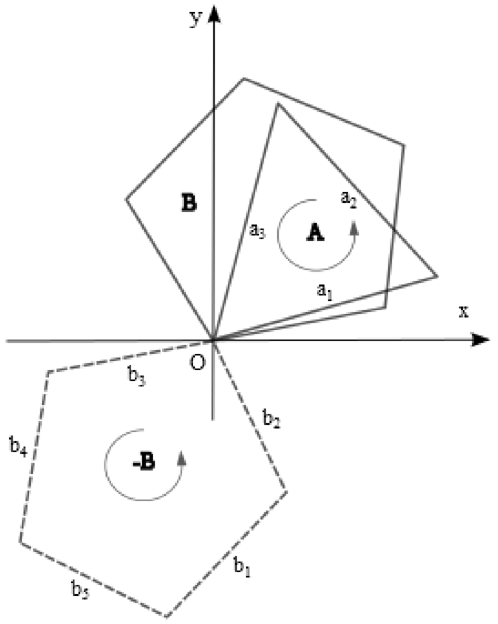

Figure 1. A and B polygons.

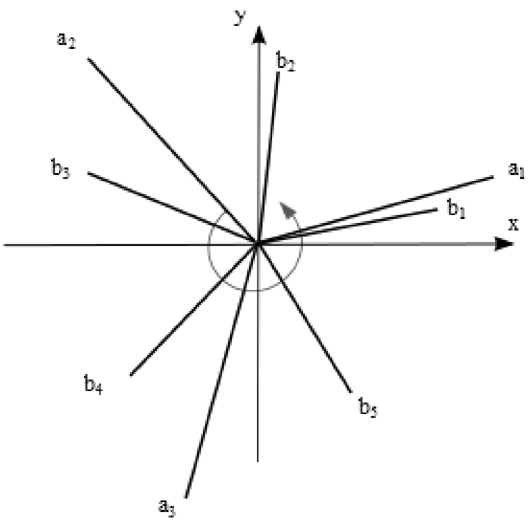

Figure 2. The a_i and b_i segments.

Once the segments sorting is done, we start the construction with the segment which has the minimum angle with horizontal axis, and then we put the following segments head to head, using the already obtained order, as you can see in figure 3. The starting point of the last segment in this order must close with the end point of the first segment. The polygon made in this way is the Minkowski sum for the considered polygons A and B. It must be mentioned that the Minkowski sum is invariant according to the position of polygons A and B. Also, we must mention that if polygon A overlaps polygon B, then the Minkowski sum polygon includes the origin of the coordinate system xOy.

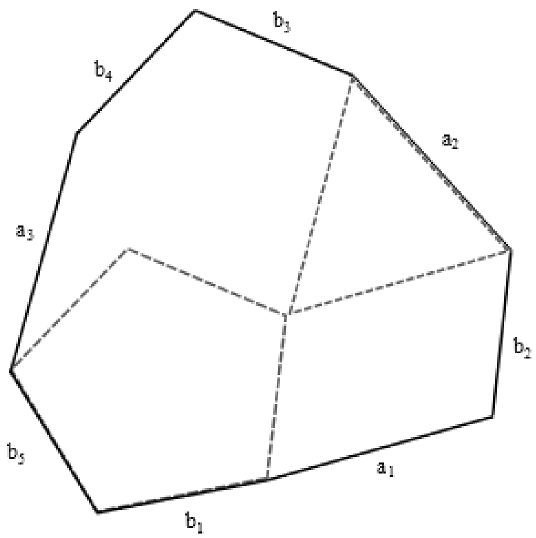

Figure 3. The Minkowski sum polygon.

2.2 Concave polygons

This time we consider the A profile as a concave polygon. The segments of both polygons A and B, must be read counterclockwise. In a cartesian coordinate system, the angles value, according to the horizontal axis, of these segments have to be calculated (see fig.4).

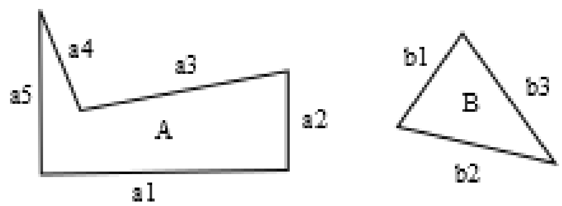

Figure 4. A and B polygons.

The placement of these segments on the Minkowski sum contour is made as in the previous case, with the important observation that when we arrive to the cavity, there will be major modifications.

So, we start with a1 segment (see fig.5), then we continue with a_2, b_3, a_3 and here we have the cavity at A polygon. As a consequence, to go from a_3 to a_4, we go again to b_3 (b_3 changes the sign), then from a_4 to b_1 and again b_3, and finally to a_5, b_2 toward the starting point of a_1 segment.

The figure 5 represents a Ghosh diagram (Ghosh, 1991). This diagram shows the placement order of a_i, b_i segments, head to head, in order to get the polygon of Minkowski sum as we see in figure 6. To complete the construction of the Minkowski sum polygon, the intersection points between segments must be calculated, and the inner segments have to be removed (see fig.7).

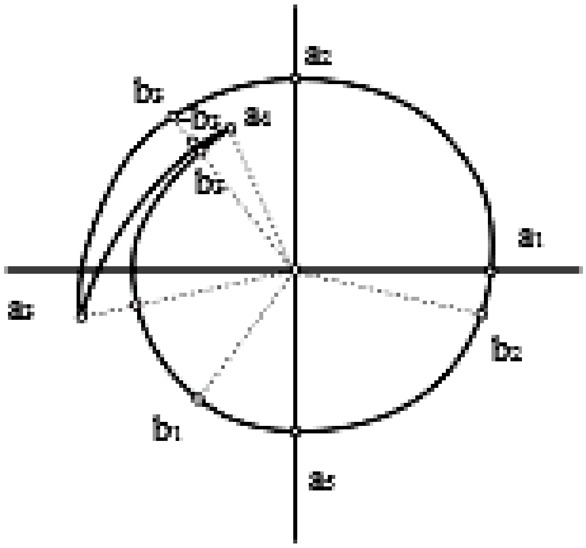

Figure 5. Ghosh diagram for A and B polygons.

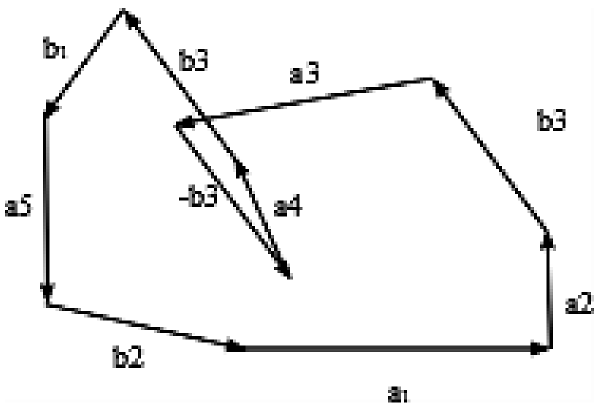

Figure 6. The Minkowski sum drawing.

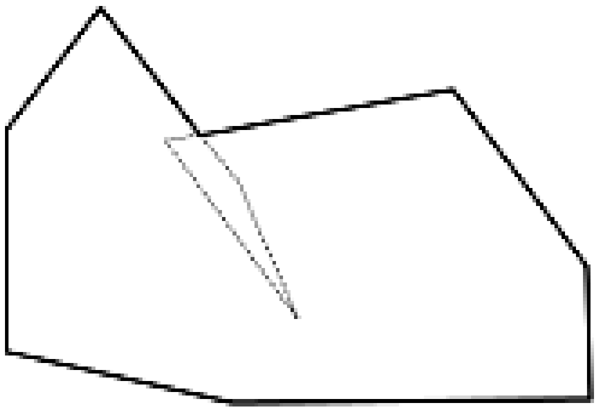

Figure 7. The Minkowski sum polygon.

Starting from this method, multiple reserches amend some special situations which might appear when we are drawing the Minkowski sum (Bennell et al, 2001), or avoid some degenerate situations such as interlocking cavities and holes (Burke, 2007).

3 OPTIMAL NESTING METHODS

3.1 The inclusion of irregular shapes into rectangles of minimum area

The method is simple, easy to use and program on a computer (Adamovitz and Albano, 1976), (Martin and Stephenson, 1988). The rectangles are easy to assemble for an optimal utilization coefficient. But we have always a difference between the rectangle area and stamping profile area which becomes a waste material.

3.2 The inclusion of irregular shapes into polygons that can be easy assembled

This variant does not differ significantly from the previous method and suffer the same shortcomings. The only positive aspect is that the new variant widens the polygonal variety that includes the stamping profiles (Chow, 1979).

3.3 Incremental orientation

Firstly, we chose an initial position of the stamping profile. For this position, the necessary width of the material strip and pitch are calculated. For these values of the strip width and pitch, the utilization coefficient is calculated. Then, the stamping profile is rotated with an angular value (the angular increment). All the calculations concerning the material strip width, pitch and utilization coefficient are repeated. The incremental rotation is continued until the profile makes an 180^{o} rotation. Due to the symmetry, there is not necessary 360^{o} angular rotation. Finally, it can be chosen that profile position with maximum value for material utilization coefficient (Nee, 1984), (Prasad et al, 1995). The idea is good, but requires a big number of calculations, depending on the incremental angular value for profile rotation. In addition, is possible that real optimal value of angular rotation to fall between two successive values of the chosen increment. Also, it is possible to find that the necessary strip width differ from the standardized material width. In this case, additional costs will be involved.

3.4 Optimal orientation using Minkowski sum

Let's consider a stamping profile which has a shape of type L, called polygon A. In a cartesian system xOy, let's draw the symmetric of A to the system origin. Let's call this new polygon as –A. We chose now a reference point on –A. Then, we move the polygon –A around polygon A in an orbital motion. During this motion, the reference point of –A will follow the contour of polygon A (see fig.8).

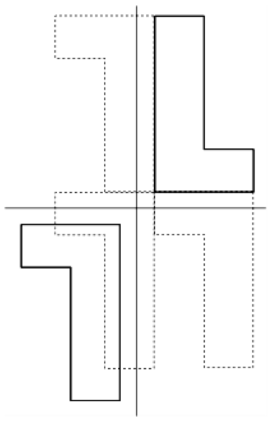

Figure 8. The orbital motion of –A around A.

The surface covered by polygon –A during the orbital motion around polygon A, corresponds to Minkowski sum polygon (see fig.9).

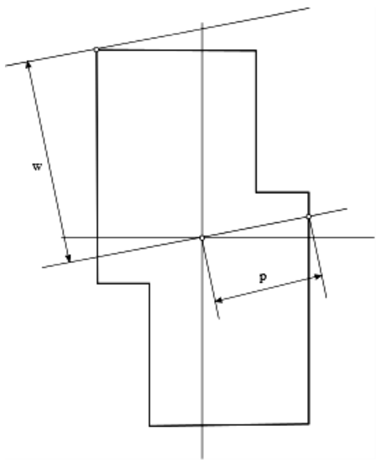

Figure 9. The Minkowski sum polygon.

The origin of the cartesian system is a symmetric point for Minkowski sum polygon. The mobile line which pass this point and has a specific inclination according to the horizontal axis, intersect the Minkowski sum polygon in a specific point (see fig.9). The distance measured from the origin of the cartesian system to this point is the pitch, denoted p, in figure 9. In the same time, the measured distance from the mobile line to the maximum value of y coordinate, on the contour of Minkowski sum, represents the minimum width of the strip material, denoted w, in figure 9.
We can chose some position of the mobile line as in figure 10, and for this position we can calculate the pitch and the minimum strip width. The necessary area of the material is equal to the product between p and w. The utilization coefficient is obtained by dividing the area of the stamping profile to area of necessary material. While this value becomes close to unit, the material loss becomes smaller. For the ideal case, this value is 1, but this is met in practice only for particular cases. The position of the mobile line from figure 11 corresponds to the situation with minimum pitch value.

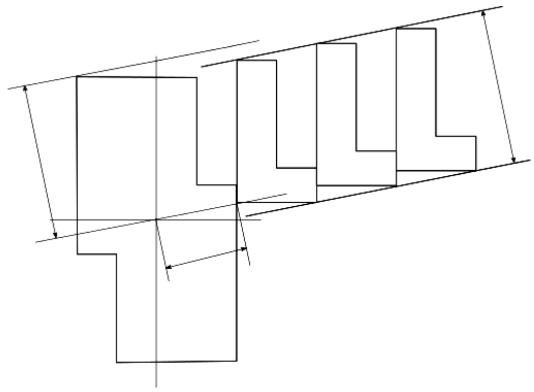

Figure 10. The profile orientation for some angular position of the mobile line.

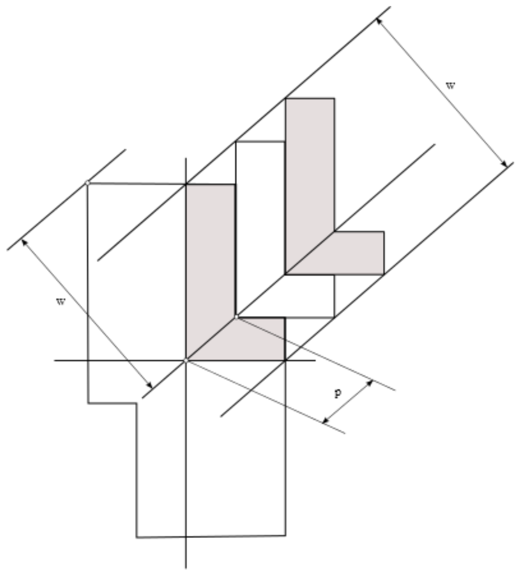

Figure 11. The profile orientation for minimum pitch value.

If we suppose that this position is optimal (i.e. the rate between the profile and minimum strip width areas has the biggest value), then we are tempted to think that there is not any other alternative to improve the material utilization coefficient. However, there are situations in which specific features of a polygonal profile allow to get a better solution than that depicted in figure 11.The idea is based on the statement that when area of the profiles in a specific position does not significantly differ from the area of their convex hull, then the material loss will be negligible.

4 THE PROPOSED IMPROVEMENT

For the particular case of an „L" type concave polygon, we build its symmetric from the origin of the coordinate system. Using an orbital motion (see fig.12), we seek that position of the symmetric polygon (case f) where the above statement is fulfilled. Once this position found, optimal positioning so that the utilization coefficient is optimal, becomes a simple matter. It must be mentioned that this simple idea can be applied to convex polygons also.

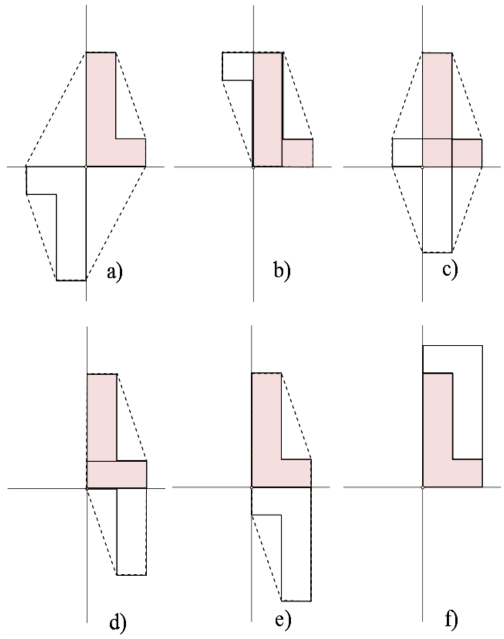

Fig. 12. Different cases in orbital motion.

In the following lines, a computer program, in pseudo-code is listed:

```
Calculate the area of A polygon
// For each vertex of polygon B = -A
for ij = 1:dimA
   for j = 1:dimA
   // calcuate the distance to translate B
      distx = xA(j)-xB(ij);
      disty = yA(j)-yB(ij);
      for i = 1:dimB
         xBprim = xB + distx;
         yBprim = yB + disty;
      end
      for i=1:dimA
         xH(dimA+i) = xB(i) + distx;
         yH(dimA+i) = yB(i) + disty;
      end
      Calculate the convex hull of A and B
      Calculate vertex number of convex hull
      Calculate area of the convex hull
      Calculate A ∩ B
      Calculate area of I = A ∩ B
      // if A and B does not overlap
      if ariaI == 0
         rate = ariaCH/(2*ariaA);
      end
      if rate < rateOptim
         rateOptim  = rate;
         xAoptim = xA;
         yAoptim = yA;
         xBoptim = xBprim;
         yBoptim = yBprim;
      end
   end
end
```

5 NUMERICAL TESTS

To exemplify the above concepts, two polygonal profiles P_1 and P_2 were chosen. First profile, P_1 is "L" type, while the second P_2, is a triangle, so is convex. The (x,y) coordinates of both profiles are listed in Table 1.

Table 1. The profiles Coordinates

Profile/ Vertex		v1	v2	v3	v4	v5	v6
P1	x	0	9	9	2	2	0
	y	0	0	2	2	4	4
P2	x	0	3	6			
	y	0	0	2			

The optimal orientation of P_1 and P_2 profiles were calculated based on Minkowski sum. The lateral bridge and the bridge between stamped profiles are considered included into the profile area. The numerical results of the optimization program which find the optimal orientation of the profiles on the material strip, based on Minkowski sum, are presented in Table 2.

Table 2. The calculated parameters

Parameter	P_1	P_2
Optimal strip width [mm]	9.83	1.66
Optimal strip pitch [mm]	2.93	3.71
Rotational angle [deg.]	45	213
Strip area [mm^2]	27.8	6.0
Profile area [mm^2]	22.0	3.0
Utilization coefficient [%]	0.79	0.50

The numerical results from table 2 were obtained based on the principles shown in paragraph 3.4. In this table the value of the rotational angle indicates how much the profile will be rotated according to its initial position.

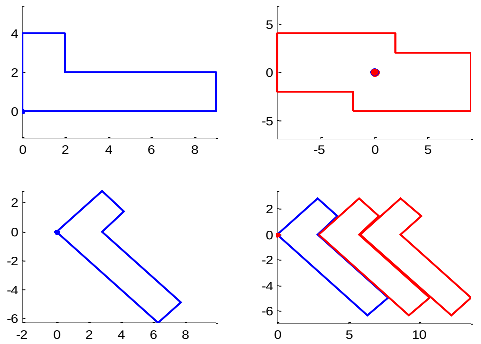

Figure 13. Optimal position of the "L" profile on the material strip.

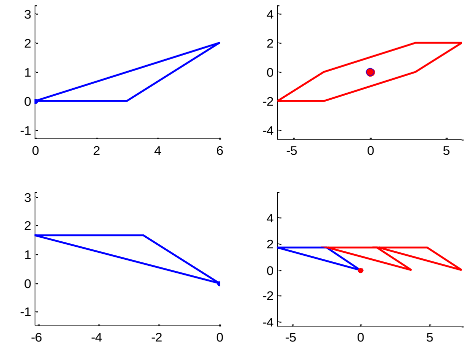

Fig.14. Optimal position of a triangle on the material strip.

Figures 13 and 14 display the initial position of the profile (top-left), the shape of Minkowski sum (top-right), the profile rotated in optimal position (bottom-left) and some profiles arranged according to their position on the material strip (bottom-right). If we apply this principle at the "L" profile accompanied by a second one in a proper position (see fig.12, f), for some particular size of this polygon, a maximum utilization coefficient can be achieved (i.e. $C_{ut} = 1$). The same situation happens for P_2 polygon, as we can see in figure 15.

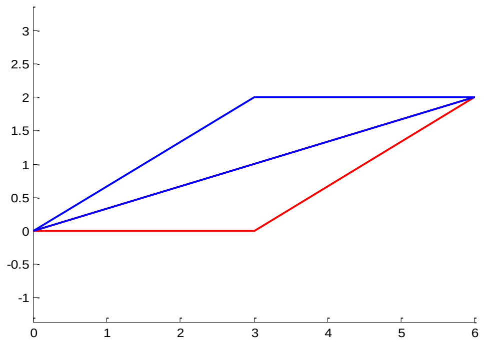

Fig. 15. Optimal position for P_2 polygon.

6 CONCLUDING REMARKS

This research paper presents a very simple method to improve the design of stamping tools design for some particular situations. The connection of two polygons of the same shape is automatically made by a computer program described in paragraph 4. However it must be specified that the computer program does not establish a maximum utilization coefficient for any "L" shape polygon or convex profile; this value depends mainly on the shape and of considered polygons. But, for some polygons with particular shape and dimensions, it is true that this method conduct to better results than the method based only on Minkowski sum. A good option is the use of both ideas in conjunction.

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