POSITION CONTROL OF AN ANTHROPOMORPHIC ROBOT ARM USING REDUNDANCY CIRCLE

Mihai CRENGANIS[1], Octavian BOLOGA[1]

ABSTRACT: This paper focuses on Simulink kinematic model of an anthropomorphic redundant robotic structure with seven degrees of freedom and a workspace similar to human arm based on the redundancy circle resolution. Another method for resolving the redundancy of a seven degrees of freedom robotic arm using Simulink is presented. The kinematic analysis and 3D virtual simulation share similar results.

KEY WORDS: kinematics, redundancy circle, seven DOF, robot arm.

1 INTRODUCTION

With the development of robotics and industrial robots various mechanical systems that mimic living systems and the human body have been developed. Different concepts of anthropomorphic robotic arms have been built so far and they are becoming more and more technological advanced. The general structure of robots is highly dependent of the utility and the purpose for which they are built. A robotic arm with seven degrees of freedom (7 DOF) shows great potential because it gains a vast working space and a high mobility and therefore it can be used in the automotive, aerospace industry and many others. Firstly the paper describes the structure of the human arm and then the model of the kinematics of a robotic arm with seven DOF is highlighted. Finally the results of the simulations are compared with the numerical analysis and then the conclusions are expressed.

2 THE HUMAN ARM

There can be considered different parameters that can characterize certain properties of human arm, for example degrees of mobility, the magnitude and dimensions of arm movements. We can consider that the human arm represented in simplified form, acquires 3 DOF for the shoulder joint, 1 DOF for the elbow joint and 3 DOF for the hand wrist. A simplified structure of a human like robot arm with 7 DOF is shown in Figure1.

[1] "Lucian Blaga" University from Sibiu, Engineering Faculty, Department of Machines and Industrial Equipment, Emil Cioran 4, 550025, Sibiu, Romania.

E-mail: mihai.crenganis@ulbsibiu.ro

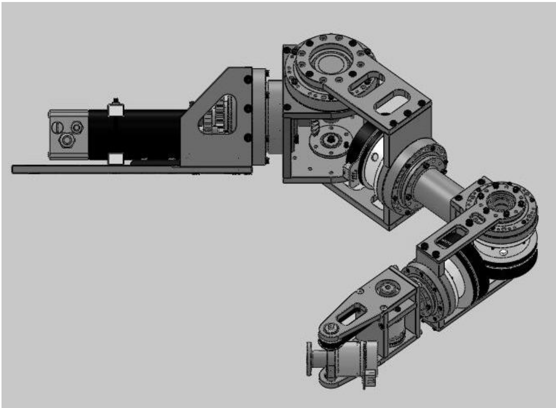

Figure 1. CAD model of the robotic arm

In Figure2 the structural scheme of the work involved in this paper is presented.

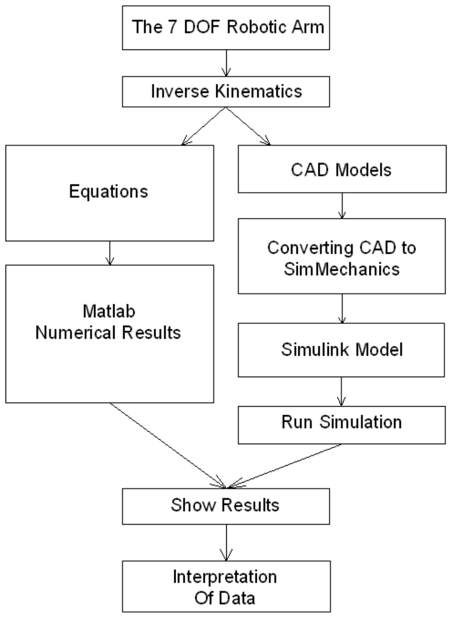

Figure 2. Structural scheme of the paper

3 THE KINEMATIC RESOLUTION USING THE REDUNDANCY CIRCLE METHOD

In the Figure 3 the structural scheme of the seven DOF robotic arm is presented.

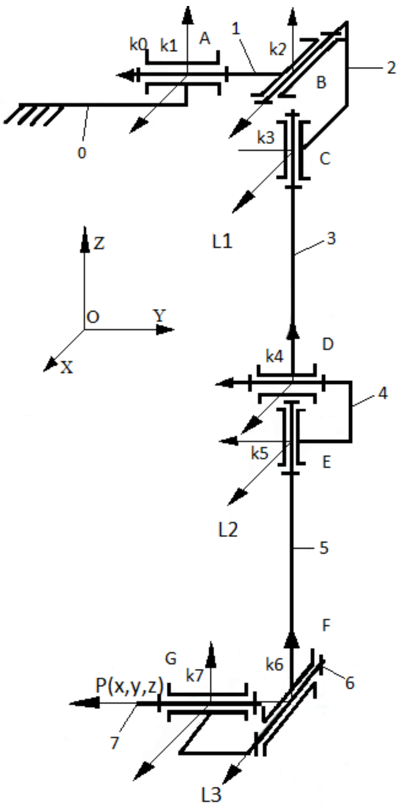

Figure 3. Kinematic structure of the robotic arm

If we attach to each element *"i"*, *(i = 0…7)* of the structure, one fixed coordinate system $k_i(O_i, x_i, y_i, z_i)$, then we can express the homogeneous transfer matrices A_i which characterize the relative movements between each element of the mechanic structure.

If we know the relative parameters θ_i *(i = 1 …7)* and the homogeneous transfer matrix form between two elements or the homogeneous transfer matrix between the coordinate systems attached to each element, we can determine the total transfer matrix between the system $k_7(O_7, x_7, y_7, z_7)$ and system $k_0(O_0, x_0, y_0, z_0)$:

$$H_{07} = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6 \cdot A_7, \quad (1)$$

were:

$$A_1 = R_y(\theta_1) = \begin{pmatrix} cos(\theta_1) & 0 & sin(\theta_1) & 0 \\ 0 & 1 & 0 & 0 \\ -sin(\theta_1) & 0 & cos(\theta_1) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (2)$$

represents the rotation between the base and the first element of the robotic arm or between k_0 and k_1 coordinate systems;

$$A_2 = R_x(\theta_2) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(\theta_2) & -sin(\theta_2) & 0 \\ 0 & sin(\theta_2) & cos(\theta_2) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (3)$$

represents the rotation between the first and the second element of the robotic arm or between k_1 and k_2 coordinate systems;

$$A_3 = R_z(\theta_3) \cdot T_z(L1) = \begin{pmatrix} \cos(\theta_3) & \sin(\theta_3) & 0 & 0 \\ -\sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & L1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (4)$$

represents the rotation between the second and the third element of the robotic arm or between k_2 and k_3;

$$A_4 = R_y(\theta_4) \cdot T_z(L2) = \begin{pmatrix} \cos(\theta_4) & 0 & \sin(\theta_4) & \sin(\theta_4) \cdot L2 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta_4) & 0 & \cos(\theta_4) & \cos(\theta_4) \cdot L2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (5)$$

represents the rotation between the third and the forth element of the robotic arm or between k_3 and k_4;

$$A_5 = R_z(\theta_5) = \begin{pmatrix} \cos(\theta_5) & \sin(\theta_5) & 0 & 0 \\ -\sin(\theta_5) & \cos(\theta_5) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (6)$$

represents the rotation between the forth and the fifth element of the robotic arm or between k_4 and k_5;

$$A_6 = R_x(\theta_6) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta_6) & -\sin(\theta_6) & 0 \\ 0 & \sin(\theta_6) & \cos(\theta_6) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (7)$$

represents the rotation between the fifth and the sixth element of the robotic arm or between k_5 and k_6;

$$A_7 = R_y(\theta_7) \cdot T_y(L3) = \begin{pmatrix} \cos(\theta_7) & 0 & \sin(\theta_7) & 0 \\ 0 & 1 & 0 & L3 \\ -\sin(\theta_7) & 0 & \cos(\theta_7) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (8)$$

represents the rotation between the fifth and the sixth element of the robotic arm or between k_6 and k_7;

We will note the elements of H_{07} like this:

$$H_{07} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}. \quad (9)$$

3.1 The inverse kinematic analysis of positions (Handra-Luca, 1996), (Deepak,1996)

To resolve the inverse kinematics problem we need to start from the fact that we know the position and the orientation of the end effector $x, y, z, \varphi_x, \varphi_y, \varphi_z$ in reference to the fixed coordinate system and we need to determine the relative positions between robot's elements. Therefore we wish to determine the relative parameters θ_i between elements which represent the rotation in the kinematic couplings. In this case *(i = 1 .. 7).*

Since this type of robot has a degree of mobility in excess of six possible in the 3D space we impose an additional condition to solve the system of equations. With this constraint one of the angles θ_i is considered known.

The steps for solving the inverse kinematic problem are:

The transfer matrix which characterize the end effector position and orientation in reference to a fixed coordinate system $k_0(O_0, x_0, y_0, z_0)$ has to be created, based on the absolute parameters $x, y, z, \varphi_x, \varphi_y, \varphi_z$;

The wrist position x_{im}, y_{im}, z_{im} has to be determined;

Knowing the wrist position and orientation we will determine the relative parameters θ_i, *(i=1...7)*;

We will consider these notations:

P (x, y, z) is the end effector position in reference to the fixed coordinate system, this section has attached the coordinate system $k_7(O_7, x_7, y_7, z_7)$

L1 - is the arm length;

L2 - the length of the forearm;

L3 - the length from *P (x, y, z)* to the wrist.

The transfer matrix which expresses the position and orientation of the end effector in reference to a fixed coordinate system $k_0(O_0, x_0, y_0, z_0)$ is composed.

The transfer matrix is denoted H_{07} and consists of the product of all homogeneous transfer matrices that characterize the end effector position and orientation in reference to a fixed system of coordinates:

$$H_{07} = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6 \cdot A_7 \quad (10)$$

were:

$$T_x = \begin{pmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (11)$$

represents the translation with *X* dimension along the *OX* axis of the k_0 coordinate system;

$$T_y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (12)$$

represents the translation with *Y* dimension along the *OY* axis of the k_0 coordinate system;

$$Tz = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (13)$$

represents the translation with *Z* dimension along the *OZ* axis of the k_0 coordinate system;

$$R_{\varphi x} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(\varphi x) & -sin(\varphi x) & 0 \\ 0 & sin(\varphi x) & cos(\varphi x) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (14)$$

represents the rotation with *φx* degrees around the *OX* axis of the k_0 coordinate system;

$$R_{\varphi y} = \begin{pmatrix} cos(\varphi y) & 0 & sin(\varphi y) & 0 \\ 0 & 1 & 0 & 0 \\ -sin(\varphi y) & 0 & cos(\varphi y) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (15)$$

represents the rotation with *φy* degrees around the *OY* axis of the k_0 coordinate system;

$$R_{\varphi z} = \begin{pmatrix} cos(\varphi z) & -sin(\varphi z) & 0 & 0 \\ sin(\varphi z) & cos(\varphi z) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (16)$$

represents the rotation with φz degrees around the *OZ* axis of the k_0 coordinate system.

The matrix that determines the position of the hand wrist is shown below:

$$H_{im} = H_{07} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} \quad (17)$$

We will note:

$$(a_{14} \cdot a_{24} \cdot a_{34} \cdot a_{44})^T = H_{im} \cdot (0 \quad 0 \quad 0 \quad 1)^T \quad (18)$$

The distance between the hand wrist position and $k_0(O_0, x_0, y_0, z_0)$ is:

$$p = \sqrt{a^2{}_{24} + a^2{}_{24} + a^2{}_{34}} \quad (19)$$

The angle θ_4 is determined using the expression:

$$\theta_4 = \pi \pm \arccos\left(\frac{L1^2 + L2^2 - p^2}{2 \cdot L1 \cdot L2}\right) \quad (20)$$

To determine the other angles, we have used the redundancy circle method. We know by fact that even if the hand wrist is fixed the elbow joint manages to describe a circular trajectory around the line segment from *O* to *B*, like shown in Figure 4. This is the most efficient method for solving the inverse kinematics problem of a serial and redundant robotic arm.

The next objective is to determine the position and orientation of the elbow joint. To do that we need to use the *Roll*, *Pitch* and *Yaw* angles starting from k_0 coordinate system. In this case we have used α, β, θ. The last one is the most important because this angle will be used to resolve the entire

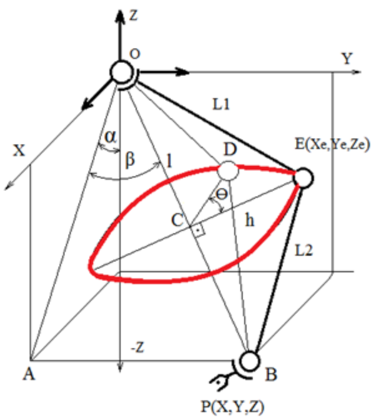

Figure 4. Redundancy circle method for inverse kinematics

inverse kinematics of the robotic arm. To be short, the input data to the inverse kinematics equations are the position and orientation of the end effector *X*, *Y*, *Z*, *Rφx, Rφy, Rφz* and the θ angle. The θ angle is zero when the *OBE* triangle is perpendicular onto *YOZ* plane.

After we have determined the distance from the wrist to the shoulder *l*, we can compute the area A_r and height *h* of the *OBE* triangle, using the Heron's formula. Starting from k_0 we can reach the elbow position using some simple homogenous transformations like:

$$He = R_x(\alpha) \cdot R_y(\beta) \cdot T_z(a) \cdot R_z(\theta) \cdot T_y(h) \quad (21)$$

where:

He, represents the entire homogenous transfer matrix that characterizes the position and orientation of the elbow joint;

α, β, θ, represent the *Roll*, *Pitch* and *Yaw* angles, the orientation of the elbow joint;

a, represents the projection of *L1* onto *l* and it is determined using Pythagora's formula in the *OCE* triangle;

h, is the height of the *OCE* triangle and *OBE* triangle.

From *He* matrix we can determine:

$$\theta_1 = atan2(He(1,1), He(3,1)) \quad (22)$$

Knowing that:

$$He = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \quad (23)$$

we have the following system of equations:

$$A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a_{i,4} \\ 1 \end{pmatrix} \quad (24)$$

If we multiply to the right with the inverse matrix of A_1 it results the following system of equations:

$$A_2 \cdot A_3 \cdot A_4 \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} b_{i,4} \\ 1 \end{pmatrix} \quad (25)$$

Because θ_4 is known it results from the first equation of the system (25):

$$\theta_3 = arccos\left(\frac{b_{14}}{L2 \cdot sin(\theta_4)}\right) \quad (26)$$

results from the second equation of the system.

To determine the angles θ_5, θ_6, θ_7, we consider the following system:

$$A_5 \cdot A_6 \cdot A_7 = A_4^{-1} \cdot A_3^{-1} \cdot A_2^{-1} \cdot A_1^{-1} \cdot H_{im} \quad (27)$$

Also we will note:

$$A_4^{-1}\cdot A_3^{-1}\cdot A_2^{-1}\cdot A_1^{-1}\cdot H_{im}=\left(m_{i,j}\right) \qquad (28)$$

From the equality of the two matrices the following results:

$$\begin{cases} \theta_6 = \arcsin(m_{32}) \\ \theta_7 = -\arctan2(m_{31}, m_{33}) \\ \theta_5 = -\arctan2(m_{12}, m_{22}) \end{cases} \qquad (29)$$

3.2 The inverse kinematic analysis of velocities

In this case it is considered as known the laws of variation of the absolute speed of the end effector $\dot{X}=(\dot{x},\dot{y},\dot{z},\dot{\varphi}_x,\dot{\varphi}_y,\dot{\varphi}_z)^T$ and we need to determine the relative speeds $\dot{q}=(\dot{q}_1...\dot{q}_n)^T$ between elements.

The expression that characterizes the relationship between the velocities is:

$$\dot{q}=J^{-1}\cdot\dot{X} \qquad (30)$$

where:

J^{-1} is the inverse Jacobian matrix *J*. In case J^{-1} can't be obtained by classical procedures because the matrix *J* is not square. It is of type 6 × 7. Thereby we will calculate the pseudo-inverse matrix J^*.The matrix is determined by the following relationship:

$$J^*=(J^T\cdot J)^{-1}\cdot J^T \qquad (31)$$

In this way the expression (30) becomes:

$$\dot{q}=J^*\cdot\dot{X} \qquad (32)$$

4 VALIDATION OF EQUATIONS USING SIMULINK

To obtain numerical results for the kinematics analysis, the equations presented above needed to be tested, that for were written in MATLAB and then the code was implemented in Simulink. The implementation was done in a MATLAB Function block, see Figure 5. The proposed equations were tested using the direct kinematics of the robotic arm and the reached positions and orientations were the same as the ones used as input for the inverse kinematics. To validate the inverse kinematics equations for a 7 DOF manipulator we have modeled a robot manipulator in SolidWorks and then we have converted the virtual robotic arm into a SimMechanics - Simscape model. The solutions to the inverse kinematics based on the SimMechanics model were then compared with the solutions based

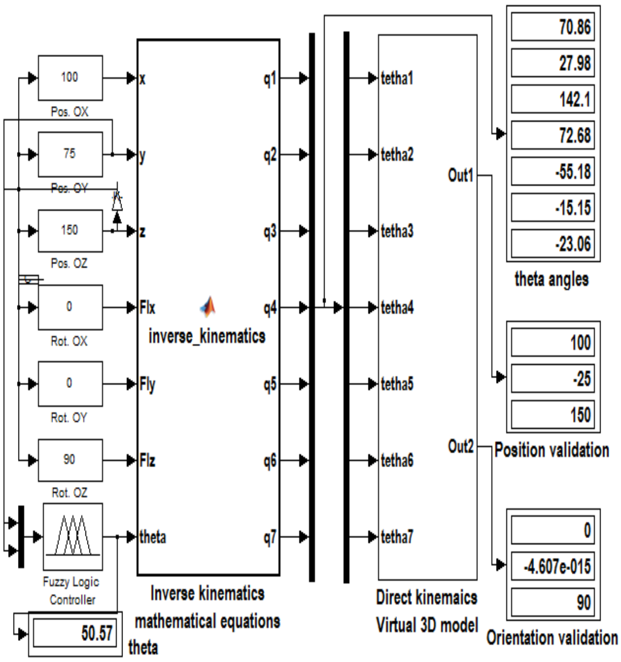

Figure 5. Block diagram of inverse kinematics

on the mathematical equations previously developed and we have obtained similar results. The SimMechanics robotic arm is presented below in Figure 6.

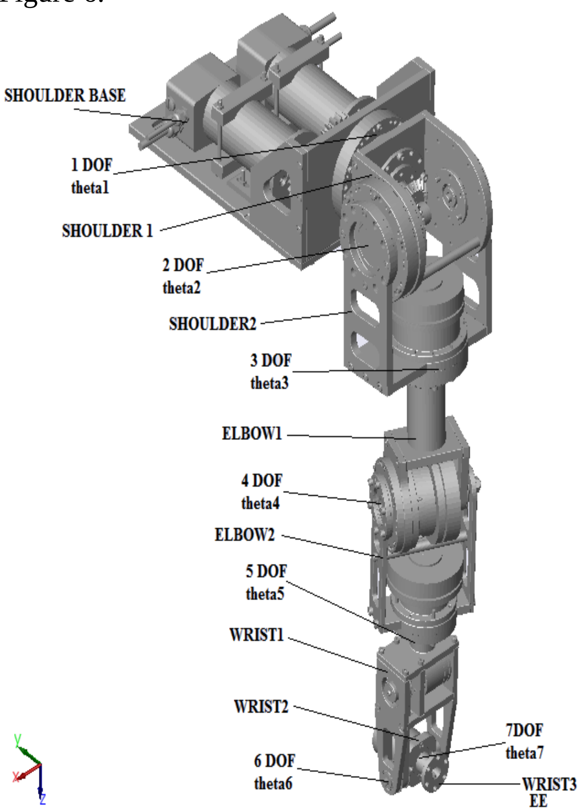

Figure 6. SimMechanics model of robotic arm

When virtual cinematic elements of the robotic arm are operated using virtual actuators placed in kinematic couplings various information regarding the position and orientation of the robotic arm are collected using virtual sensors placed in the kinematic couplings or attached onto the endeffector. SimMechanics is an environment that uses block diagrams and it is used in the modeling and simulation of the movement of rigid mechanisms based on Newton's dynamic forces. This environment is taking into consideration the size of the rigid bodies, their masses, possible movements of bodies, different geometric constraints and the relative coordinate systems. SimMechanics working environment library is divided into two different groups and they contain aspects such as weight, position and orientation, namely the relations between the different coordinate systems attached to bodies. Also it includes blocks that define the kinematic couplings of bodies such as the degree of mobility. The first step in creating SimMechanics virtual models, is the designing of the CAD robotic arm into SolidWorks.

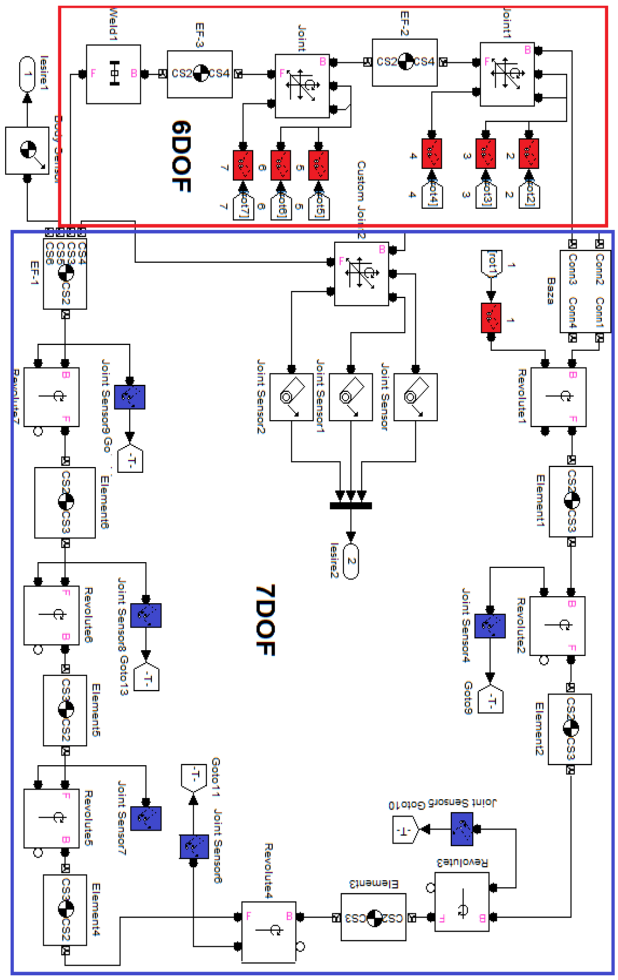

Figure 7. SimMechanics inverse kinematics

5 ACKNOWLEDGEMENTS

Mathematical equations were developed based on the kinematic structure but also after consultation of the scientific literature.

The chosen analytical method for solving inverse kinematics problem was effective, accurate and easy to implement.

The mathematical equations based on the redundancy circle described by the 7 DOF robotic arm elbow joint led to stability in movement and no impossible to reach positions for the end effector.

Mathematical equations underlying the robot kinematics have been implemented in MATLAB which led to their effective resolution.

SimMechanics – Simulink environment was used to run the simulations and to validate the equations.

This 7 DOF robotic arm shows a great working space, highly mobility and we are hoping to use this additional degree of freedom in complex applications.

6 REFERENCES

►Angeles J. et al., Fundamentals of Robotic Mechanical Systems: Theory, Methods, and Algorithms 3rd Edition. Springer-Verlag, 2006.

►Antonelli G., Indiveri G. and Chiaverini S., Prioritized closed-loop inverse kinematic algorithms for redundant robotic systems with velocity saturation. Proceedings of IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, 5892–5897, 2009.

►Brisan C., Hiller M., Modeling of Anthropomorphic Robots. IEEE International Conference INES, 601-606, 2004.

►Craig, J., “Introduction to robotics”. Reading, MA: Addison-Wesley, 1989

►Deepak, T., “Real-Time Inverse Kinematics of the Human Arm”, Presence. Vol. 5. No.4. Fall 1996, 393-401 by the Massachusetts Institute of Technology, 1996

►Handra-Luca, V., Mătieş V., Brişan, C., Tiuca, T.,” Roboţi Structură Cinematică şi Caracteristici”, Ed. Dacia, Cluj-Napoca, 1996, pag. 37-48

►Sciavicco L. and Siciliano B., A solution algorithm to the inverse kinematic problem for redundant manipulators. IEEE J. of Robotics and Automation, 303–310, 1988.