CUTTING TOOLS FOR GENERATING OF ROTORS COMPRESSORS

Camelia POPA[1], Virgil Gabriel TEODOR[2], Nicuşor BAROIU[2] and Silviu BERBINSCHI[1]

ABSTRACT: The compressor rotors usually are helical surfaces with constant pitch and are composed of crossing profiles. Frequently, for repair operations, the reconstruction necessity occurs for one or both of the worms, drive and driven, from the helical compressors gear. The theoretical helical surface of the worm is being substituted by an assembly of helical lines which together with crossing profiles forms points cloud resulted from measuring leads to a polyhedral expression of the flank rotor. Numerically, this surface type is expressed by a coordinate array which shows its discrete image. The profiling of cutting tool bounded by a revolution surface reciprocally enveloping with the substitutive surface of the helical one represents a special problem. In this paper is proposed an algorithm for polyhedral expression of the helical surface previously determined by reverse engineering methods and an algorithm for the determination of the specific enveloping condition at contact with a discrete surface. It is presented an example for a compressor rotor measured on a 3D measuring machine, the algorithm for the transformation of the gathered points cloud in a surface with polyhedral expression. Given these conditions there were determined the enveloping condition and the axial section of the side mill.

KEY WORDS: helical compressor, reverse engineering, side mill profiling, polyhedrons method.

1 INTRODUCTION

Twin-screw compressors are the core component of cooling and air conditioning systems. They are widely used in medical instruments, food processing machines, and vehicle power systems.

Several important clearances lead to fluid leaks, including clearances in the inter tooth contact band, clearances between the rotor suction/discharge ends and the housing end plugs, clearances between the rotor tooth tip and the housing wall, and the blow hole formed between the tips of the teeth of the male and female rotors and the cavity wall (Hsiao et al, 2012).

Numerical design tools are established and integrated to calculate the compressor performance, rotor temperature distributions, and thermal deformations of rotor for a commercial refrigerant (Hsieh et al., 2012).

Traditionally, volume and area computations are done either by utilizing close form analytical formulation, or by numerical area integration.

In these approaches, the areas are divided into zones with simpler geometry; areas of these zones are computed by numerical integration and added together (Ignatiev, 2012). The groove flank is the envelope surface corresponding to the curved surface of the tooth flank. Section profile of the new tooth flank is a curved line, which could be elliptical, hyperbolic or involute (Popa et al., 2014).

Screw compressor rotors are machined today by grinding or milling, usually in two stages; roughing, when the work piece is machined to its approximate size, and finishing, when the rotors are machined to their final dimensions. The theory of enveloping surfaces is used to calculate the relative motion between each point of the tool and the rotor during the cutting process. A theory of surface enveloping was applied to simulate the cutting process as the tool conducts polynomial feed motion considering its cutting edge (Hsieh et al., 2012).

2 GENERATING TOOLS PROFILING

The profiling of tools designed for helical compressors rotors generation, starting from the physical models of these, constitute a special aspect of the machining of this type of parts. The problem of surface description may be solved using measuring machines which allow a rigorous determining of the axial section of the rotor and, so, the knowledge about discrete form of the helical

[1]"Dunărea de Jos" University of Galaţi, Mechanical Engineering Department, Domnească 111, 800002

[2]"Dunărea de Jos" University of Galaţi, Depart. of Manufacturing Engineering, Domnească 111, 800002

E-mail: lacramioara.popa@ugal.ro; vteodor@ugal.ro; nbaroiu@ugal.ro; silviu.berbinschi@ugal.ro

surface as a coordinate array (Berbinschi et al., 2013).

A solution of this problem may be found by measuring a 3D model of a pump rotor onto a horizontal profile projector. With the Starrett Optical profile projector the crossing section of the rotor was measured (fig. 1).

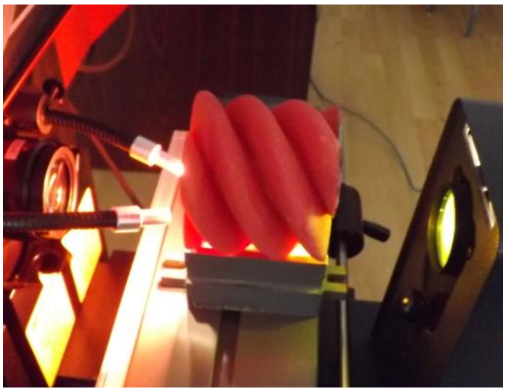

Figure 1. Measuring of rotor crossing section onto Starrett Optical profile projector

In a reference system, the composed crossing profile of the rotor can be expressed as a points array,

$$S_T = \begin{Vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_n & y_n \end{Vmatrix} \tag{1}$$

where n is big enough for all the areas of the rotor.

3 THE HELICAL SURFACE EXPRESSED IN DISCRETE FORM

If the discrete helical surface is known as coordinate array, it is possible to describe the helical movement:

$$X = \left[\omega_3^T (k\Delta\varphi) \left(\| x_i \quad y_i \| \right)_{i=1...n} + p(\Delta\varphi k) \right]_{k=1...m} \tag{2}$$

with $\Delta\varphi$ angular increment arbitrary chosen for the helical motion, k integer, k=1,2,…, m and p helical parameter of the rotor.

In this way, the helical surface is known as a coordinate array:

$$S_\Sigma = \left(\| x_i \quad y_i \quad z_k \|_{i=1 \div n} \right)_{k=1 \div m}. \tag{3}$$

By this substitution form, the helical surface, known in discrete form, is replaced with a quadrangle, ($M_{i,k}$, $M_{i,k+1}$, $M_{i-1,k+1}$, $M_{i-1,k}$), i=1…n; k=1…m.

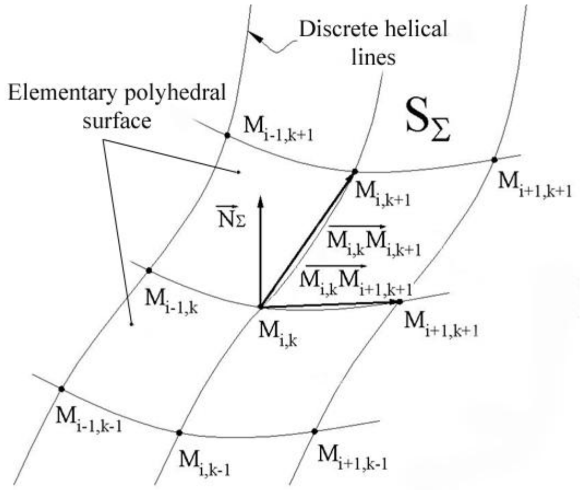

Figure 2. Substitutive polyhedral surface

It is accepted that the total number of these quadrangles constitute the measured helical surface.

If the $\Delta\varphi$ increment, according with Eq. (2), is relatively small and m and n from Eq. (3) are big enough, this expression form is a rigorous approximation of the helical surface.

4 SIDE MILL TOOL'S PROFILING ALGORITHM

An algorithm for side mill tool's profiling, known in discrete form, is needed. The reference systems are defined:

XYZ is the reference system where the helical surface is defined;

$X_1Y_1Z_1$ – the reference system associated with the side mill;

a – the distance between the axis of helical surface with constant pitch and the axis of the side mill. This value is technological established.

The relative position of the two reference systems is given by transformation:

$$\| X_1 \|^T = \omega_1(\alpha) \cdot \| x - a \quad y \quad z \|^T. \tag{4}$$

The α angle is the inclination angle of the helix which belongs to the helical surface, on the cylinder with the R_e external radius,

$$\tan\alpha = \frac{p_e}{R_e}, \; p_e \text{ helical parameter (fig. 3).} \tag{5}$$

The contact between the revolution surface and the helical surface, expressed in discrete form, is determined based on the Nikolaev's theorem:

$$\left|\vec{N}_{\Sigma M_{ik}} \times \vec{A} \times \vec{r}_{1M_{ik}}\right| \le \varepsilon \quad (6)$$

where:

$\vec{N}_{\Sigma M_{ik}}$ - is the normal to the M_{ik} to the S_Σ helical surface;

$\vec{A}$ - is the versor of the side mill axis;

$\vec{r}_{iM_{ik}}$ - the position vector of the current point $(M_{ik})_{i=1\ldots n,k=1\ldots m}$;

ε - arbitrary chosen.

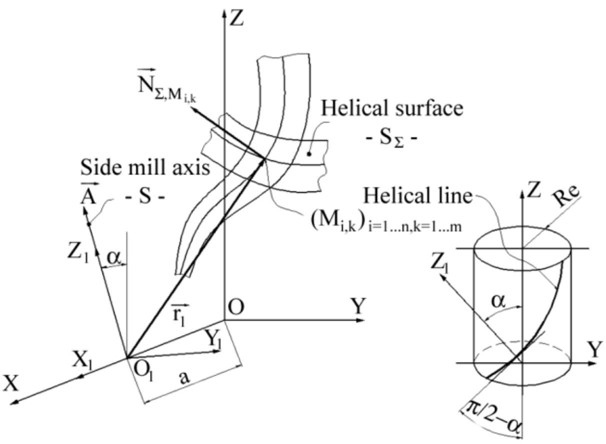

Figure 3. The side mill axis position regarding the helical surface. References systems

It is defined:

- the normal to the S_Σ surface in the current point M_{ik}, (fig. 2):

$$\vec{N}_{\Sigma,M_{ik}} = \overline{M_{ik}M_{i+1,k}} \times \overline{M_{ik}M_{i,k+1}}; \quad (7)$$

- the versor of the $\vec{A}$ axis of the side mill, in the XYZ reference system

$$\vec{A} = -\sin\alpha\,\vec{j} + \cos\alpha\,\vec{k} \quad (8)$$

with α defined by (5);

- $\vec{r}_{1M_{ik}}$ position vector of the M_{ik} point from S_Σ in XYZ reference system, (9):

$$\vec{r}_{1M_{ik}} = \left(X_{ik} - a\right)\vec{i} + \left(Y_{ik}\cos\alpha + Z_{ik}\sin\alpha\right)\vec{j} - \left(Y_{ik}\sin\alpha + Z_{ik}\cos\alpha\right)\vec{k}$$

The totality of points belonging to the S_Σ surface that satisfy the condition (6), determines on that the characteristic curve C_{S_Σ}, on form:

$$C = \left\| \left(X_C\right)_{i,k} \quad \left(Y_C\right)_{i,k} \quad \left(Z_C\right)_{i,k} \right\|^T_{i=1..n,k=1..m} \quad (10)$$

which, by transformation Eq. (4) is reported to the $X_1Y_1Z_1$ reference system in form:

$$C = \left\| \left(X_{1C}\right)_{i,k} \quad \left(Y_{1C}\right)_{i,k} \quad \left(Z_{1C}\right)_{i,k} \right\|^T_{i=1..n,k=1..m} \quad (11)$$

(fig. 4), with $\left(X_{1C}\right)_{i,k}$, $\left(Y_{1C}\right)_{i,k}$ and $\left(Z_{1C}\right)_{i,k}$ from Eq. (11).

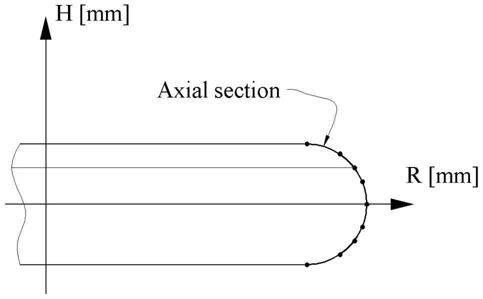

Figure 4. The axial section of side mill

The axial section of the side mill (fig. 4) is obtained for:

$$H = \left(Z_{1C}\right)_{i,k};$$
$$R = \sqrt{\left[\left(X_{1C}\right)_{i,k}\right]^2 + \left[\left(Y_{1C}\right)_{i,k}\right]^2}. \quad (12)$$

5 APPLICATION

An application example is presented for a compressor rotor, known as physical model obtained by rapid prototyping. The frontal section of the rotor was measured onto a Starrett Optical horizontal profile projector (fig. 1).

Table 1. Coordinates of the rotor crossing section

Crt. no.	X_i [mm]	Y_i [mm]
1	20.04	18.978
⋮	⋮	⋮
350	48.997	-0.61
⋮	⋮	⋮
700	18.999	-20.02

It is measured the helical surface axial pitch p_{ax} = 120 mm. By this pitch, it is determined the rotor helical parameter:

$$p = p_{ax} / 2\pi \text{ [mm]}. \quad (1)$$

For a worm with four lobes, the S_Σ array is determined using the angular pitch. For the determining of the side mill axis position the distance a = 100 mm is accepted. The

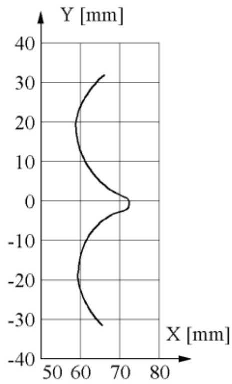

Figure 5. Side mill axial section

axial section of the side mill is determined using an in house software, see table 2, figure 5 and Eq. (13).

Table 2. Axial section of the side mill

Crt. no.	R [mm]	H [mm]	Crt. no.	R [mm]	H [mm]
1	66.023	31.898	**51**	72.352	-0.175
2	65.411	31.326	⋮	⋮	⋮
3	64.815	30.736	**97**	63.824	-29.560
⋮	⋮	⋮	**98**	64.362	-30.196
49	71.274	0.972	**99**	64.924	-30.813
50	71.952	0.511	**100**	65.511	-31.409

The screw compressor is one of the most common types of machine used to compress gasses. In the foregoing, we have presented a methodology based on reverse engineering method for profiling generating tool for the male rotor. A similar method can be applied, for the same purpose, generating side mill tool for the female rotor.

To avoiding the measurement of the transverse profile of the female rotor and description in discrete form, as in form (1), in the following we propose the determination of the cross section of the female rotor as enveloping of the male rotor front profile, in their relative motion, (fig. 6). The reference systems to which are defined the two centrodes associated to the rotors crossing sections, are: S_E for the male rotor, known in discrete form, according to (1); $S_{\xi\eta}$ - the enveloping profile to profile S_T, in rolling motion of the two centrodes, namely radius: R_{rp_1}, R_{rp_2}.

- xy is the fix reference system;
- x_0y_0 - the fixed reference system associated of the female rotor axis;
- XY - the relative system associated to the cross section of male rotor, the measured profile;
- $\xi\eta$ - the relative system associated to the cross section of female rotor.

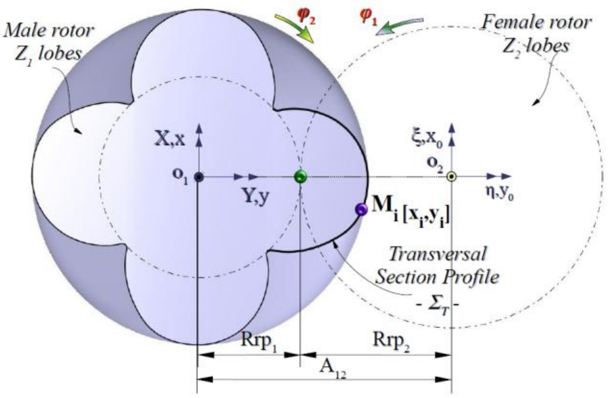

Figure 6. Centrodes associated to profiles and reference systems

The rolling radii are determined of conditions:

$$A_{12} = R_{rp_1} + R_{rp_2}, \tag{14}$$

A_{12} represents the distance between axes, measurable to the compressor housing.

The condition $R_{rp_1}\phi_1 = R_{rp2}\phi_2$, (15)

is the rolling condition, where the ratio:

$$\phi_2 / \phi_1 = z_2 / z_1, \tag{16}$$

z_1, z_2 is the number of lobes of the two rotors.

The relations (14), (15) allowed determination of rolling radius size of the two rotors, male and female. The process cinematic involves the movements:

$x = \omega_3^T(\phi_1)X_1$, for the male rotor; (17)

$x_0 = \omega_3^T(-\phi_2)\xi$, for the female rotor. (18)

Considering the relations between the two fixed reference systems

$$x_0 = x - \begin{vmatrix} A_{12} \\ 0 \end{vmatrix}, \tag{19}$$

$$\omega_3^T(-\phi_2)\xi = \omega_3^T(\phi_1)X - \begin{vmatrix} A_{12} \\ 0 \end{vmatrix}, \tag{20}$$

or

$$\xi = \omega_3^T(-\phi_2)\left[\left\{\omega_3^T(\phi_1)X - \begin{vmatrix} A_{12} \\ 0 \end{vmatrix}\right\}\right] \tag{21}$$

The transformation (21) allowed determination of the family profiles S_T (fig. 1), in the reference system associated. We can notice that profile S_T is known in discrete form, as a matrix of points coordinates, belonging to the profile measured. The envelope of family profiles S_T represents the transverse profile of the male rotor.

6 “MINIMUM DISTANCE” METHOD

“The minimum distance” theorem (Oancea, 2004), as a complementary theorem of enveloping surfaces theory, for surfaces related with a moving couple of axoides (centrodes), approves the fact that the envelope of one profile associated with a couple of centrodes in rolling movement, represents the locus of points belonging the profile to which the distance to the gearing pole, for different positions of rolling, is minimum. Thus (fig. 7), the coordinates of gearing pole, in $\xi\eta$ system, are:

$$P \begin{vmatrix} \xi_P = & -R_{rp_2} \cos\phi_2; \\ \eta_P = & R_{rp_2} \sin\phi_2. \end{vmatrix} \tag{22}$$

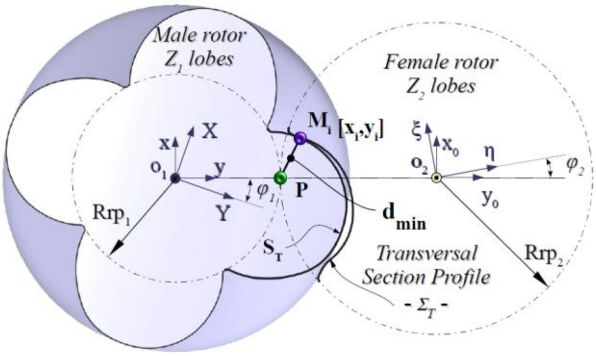

Figure 7. "Minimum distance" method

Coordinates of current point on the profile S_T, in the same reference system, are deducted from (6), for which X is the matrix of measured points on the profile S_T, see (1),

$$M_i \begin{vmatrix} \xi = & \xi_i; \\ \eta = & \eta_i. \end{vmatrix} \tag{23}$$

Distance *d* is defined by:

$$d = \sqrt{(\xi_P - \xi_i)^2 + (\eta_P - \eta_i)^2}\ , \tag{24}$$

point (ξ_i, η_i) belonging to profile S_T, in $\xi\eta$ system. Thus, all points belonging to profiles family S_T, expressed in discrete form, to which the discrete values, arbitrary, of parameter φ_i (in rolling movement) determine a minimum size of the distance (9), represents the profile of male rotor. Of principle, the transverse profile of male rotor, will be expressed through the matrix of coordinates as follows:

$$\Sigma_T = \begin{Vmatrix} \xi_1\eta_1 \\ \xi_2\eta_2 \\ \ldots\ldots \\ \xi_n\eta_n \end{Vmatrix} \tag{25}$$

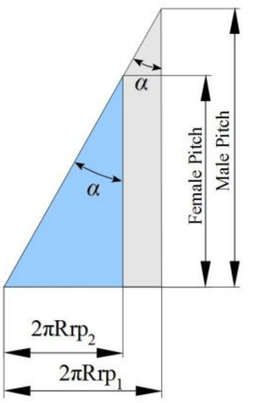

Figure 8. Axial pitch of worms

Equation (25) allows achieving the 2D model of crossing section on male rotor. To construct the 3D model of the male rotor, one has to know its helical pitch, as shown in figure 8.

The α angle is the inclination angle of the helix belongs to the helical surface of male rotor, on the cylinder with the R_e external radius, determined by measuring, as follow:

$$tg\alpha = 2\pi R_{r_1} / p_{axmale} \tag{26}$$

In the same way, for the female rotor

$$tg\alpha = 2\pi R_{r_2} / p_{axfemale} \tag{27}$$

Finally, we obtain the axial pitch of the male rotor, which along with crossing section of the rotor, will allow virtual construction of solid model of rotor. The profiling of the side mill to generating the female rotor will be achieved following the same graphic algorithm as in male rotor case.

7 APPLICATION

In what follows, there is an application for determination, using the graphic method previously described, of the peripheral surface of side mill for generating the female rotor, component of screw compressor (the male rotor crossing section have been obtain through measuring), as shown in (1).

Dimensional characteristics of screw compressor rotor:

- Gear ratio, $z_2 / z_1 = 6/4$;

- Distance between axis (measured), A_{12} = 80 mm;

- Rolling radii, R_{rp_1} = 32 mm; R_{rp_2} = 48 mm;

- Axial pitch of female rotor (according to (26) and (27)), $p_{axfemale}$ = 120 mm;

- Front profile of female rotor, obtained through the presented algorithm (table 3).

In figure 9, the solid model of the male and female rotor is shown.

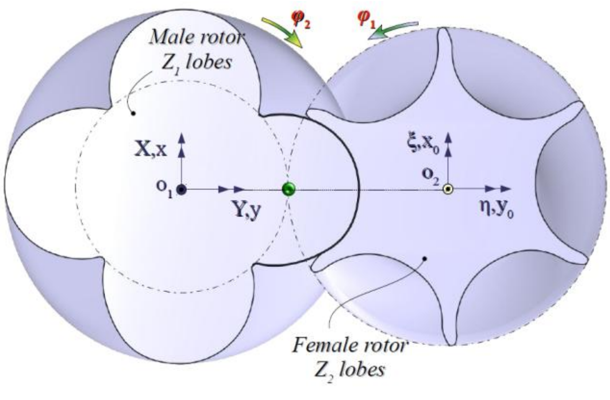

Figure 9. The solid model of male and female

Table 3. Front profile of female rotor

Point no.	Tool Profile		Point no.	Tool Profile	
	X_S [mm]	Y_S [mm]		X_S [mm]	Y_S [mm]
1	38.003	23.244	**14**	53.272	-3.204
2	37.964	23.167	**15**	53.264	-3.290
3	37.929	23.088	**16**	53.256	-3.376
4	37.900	23.007	...	...	...
5	37.876	22.924	**26**	38.721	-24.391
...	...	...	**27**	38.745	-24.473
11	53.294	-2.945	**28**	38.775	-24.555
12	53.287	-3.032	**29**	38.810	-24.634
13	53.280	-3.118	**30**	38.850	-24.710

In figure 10, the solid model of side mill with its Disc Tool is shown.

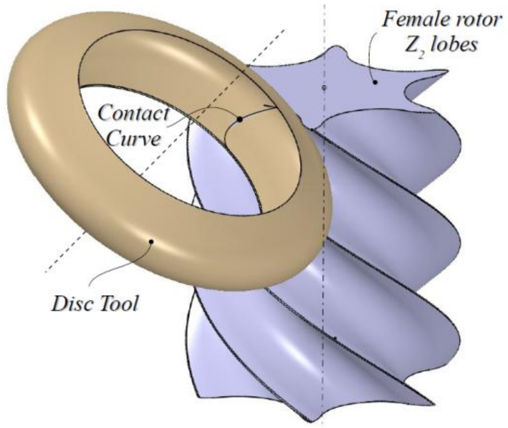

Figure 10. The solid model of side mill with Disc Tool

Table 4. Axial section of the side mill

Point no.	Tool Profile		Point no.	Tool Profile	
	X_S [mm]	Y_S [mm]		X_S [mm]	Y_S [mm]
1	48.134	16.560	**14**	68.575	3.220
2	49.623	15.447	**15**	68.944	1.314
3	51.366	14.585	**16**	68.977	0.628
4	53.127	13.762	...	...	...
5	54.907	12.979	**26**	55.090	12.496
...	...	...	**27**	53.243	13.103
11	65.393	8.009	**28**	51.414	13.760
12	66.757	6.628	**29**	49.602	14.464
13	67.840	5.016	**30**	48.066	15.466

In figure 11, the axial section of side mill is presented.

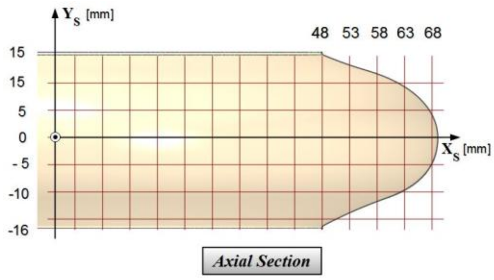

Figure 11. The axial section of side mill

8 CONCLUSIONS

The proposed algorithm for the side mill profiling for the generation of a rotor allows the determination of the tool's axial section. The rotor belonging to the assembly of a helical compressor is known by the measurement of the frontal section of the 3D model obtained by rapid prototyping.

The method can be applied for others situations, if it is possible the rigorous measurement of the frontal section.

9 ACKNOWLEDGEMENTS

The work has been funded by the Sectoral Operational Programme Human Resources Development 2007-2013 of the Ministry of European Funds through the Financial Agreement POSDRU/159/1.5/S/132397.

10 REFERENCES

► Hsiao, H.H., Wu, Y.R. and Hsieh, H.C. (2012) *Non Uniform Clearance between Rotor Surfaces and Its Effect on machine Performance in Twin Screw Compressors*, Purdue – International Compressor Engineering Conference. Paper 2087.

► Hsieh, S., Hsieh, W., Huang, C. and Huang, Y. (2012). *Numerical Analysis of Performance, Rotor Temperature Distribution and Rotor Thermal Deformation of an R134a Screw Compressor*, Purdue – International Compressor Engineering Conference, Paper 2115.

► Ignatiev, K.M. (2012). *Approach to the Numeric Geometry Analysis of Positive Displacement Compressors, its Application to a Single Screw Compressor Simulation and Verification by Experiment*, Purdue – International Compressor Engineering Conference, Paper 2070.

► Berbinschi, S., Baroiu, N., Teodor, V., Oancea, N. (2013). *Profiling Method of Side Mill for Threading Screw for Dental Implants*, Proceedings of Modern Technologies in Industrial Engineering, Advanced Materials Research, Vol. 837, pp. 22 27.

► Popa, C.L., Teodor, V.G., Baroiu, N. and Oancea, N. (2014). *Side Mill Tool Profiling for Generation of Helical Surfaces Determined by Reverse Engineering*, Proceedings of Innovative Manufacturing Engineering International Conference, IManE, May 29-30, 2014, Chişinău.

► Oancea, N. (20014). *Generarea suprafeţelor prin înfăşurare, Vol II, Teoreme complementare*, Editura Fundaţiei Universitare "Dunărea de Jos" Galaţi, ISBN973-627-106-4, ISBN973-627-170-6, Vol. II.