BAYESIAN THEORY-BASED RELIABILITY MODELING OF THE TOOL LIBRARY OF THE MACHINING CENTER

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ABSTRACT*: Under the condition of small samples, the failure rate-based reliability modelling of the tool library was carried out. This paper selects binomial distribution as the likelihood function and conjugate distribution as the prior distribution and carries out parameter estimation with the generalized-methods-of-moments to establish the pilot sample reliability model of the tool library system laboratory. This model is consistent with the actual status, signifying that it is effective.*
KEYWORDS*: binomial distribution, Bayesian theory, small sample.*

1 INTRODUCTION

As the mother machine in the manufacturing industry, the numerically-controlled machine tool is the basis of manufacturing competitiveness. According to the field experience and years of research experience, the key functional components enable the implementation of the machine tool function and guarantee the machine reliability. Therefore, the reliability of the tool library system which is an important part of the key functional component, is especially important. The tool library system is a key functional unit of the machining center, assuming the task of quick tool changing and installing (Pesinis, 2017). The Reliability level of the system is directly related to whether the machining center can quickly and qualifiedly complete the processing task (Jia, 2001; Xiong, 2006; Fiondella, 2016). It is the key factor affecting the reliability of the machining center. The reliability growth of the tool library system is a systematic project, including design, test, manufacture, usage, improvement, etc. Therefore, the study on the reliability modeling techniques of the tool system is of great significance to understand the rule of failures of the tool library system, conduct reliability evaluation and reliability test and formulate reasonable maintenance strategies (Liu, 2008; Yu, 2005; Gu, 2016; Pan, 2017).

In the process of reliability modeling of the tool library system, the case of small sample data (the quantity of subsystem failure data is rather small) is frequently encountered. At this time, if the large sample data parameter estimation and modeling method are adopted, the resulting model can generate large errors. Based on the Bayesian theory, the reliability modelling of the tool library system based on the fault rate is carried out. In this paper, the conjugate distribution was selected as the prior distribution and parameter estimation was carried out with the generalized-methods-of-moments to establish the pilot sample reliability model of the tool library system laboratory (Sarja, 2009; Kaeding, 2015; Insua, 2000; Gupur, 2011).

2 SELECTION OF PRIOR DISTRIBUTION

In view of the characteristics of the fault data of the tool library system, the conjugate prior distribution was selected (Chandna, 2016; Lin, 2017; Cha, 2017). Assume that the failure data in the tool library system reliability test are as follows: ($x_1, x_2, x_3 \ldots x_r$). Where, 1, 2......r is the serial number of the malfunction. The total number of samples is n. Then, the number (r) of units which are out of order in the sample data is subject to the binomial distribution:

$$P(r / p_i) = \binom{n}{r} p_i^r (1 - p_i)^{n-r} \quad (1)$$

Where, p_i is the failure rate of the tool library system. The conjugate prior distribution is to assume that $P=\{p(x/\theta)\}$. It is the distribution family identified by θ. $F=\{\pi(\theta)\}$ was assumed to be a prior distribution family in the parameter space of X. For any $p\in$P and $\pi(\theta)\in$F, if they can make the posteriori distribution $\pi(\theta)$ still belong to F, F is called the conjugated family of P. Any distribution $\pi(\theta)$ in F becomes the conjugate prior distribution of the parameter θ. The selection of the conjugate prior distribution is determined by the factor of θ which is contained in the likelihood function $L(\theta)=\{p(x/\theta)\}$. If the distribution having the same nucleus as the likelihood function $L(\theta)$, then conjugate prior distribution came into being. Otherwise, it does not exist. The commonly used conjugate prior distribution is shown in Table 1.

Table 1. Classification of conjugate prior distribution

Population distribution	Parameter	Conjugate prior distribution
binomial distribution	success probability	Beta distribution
posson	mean value	Gamma distribution
exponent distribution	reciprocal of mean value	Gamma distribution
normal distribution (variance known)	mean value	normal distribution
normal distribution (mean value known)	variance	inverse Gamma distribution

The conjugate distribution corresponding to binomial distribution is $Be(\alpha, \beta)$ Since beta distribution has the same form of nucleus with the binomial distribution, the posteriori distribution can also be taken as beta distribution and Beta distribution is taken as the prior distribution of the failure rate (Yang, 2015; Guo, 2017; Huang, 2016; Kan, 2016).

$$\pi(p_i):\ Be(\alpha,\beta)\propto\theta^{\alpha-1}(1-\theta)^{\beta-1} \quad (2)$$

It was assumed that the total number of sample is n, and the number of failure is x. The density function of the joint distribution is subject to binomial distribution:

$$h(x,p_i)=\binom{n}{x}p_i^x(1-p_i)^{n-x} \quad (3)$$

Based on Bayesian theory, the posteriori distribution of p_i is as follows:

$$\pi(p_i/x)=\frac{h(x,p_i)g\pi(p_i)}{\int h(x,p_i)g\pi(p_i)dp_i}=\frac{\binom{n}{x}\theta^x(1-\theta)^{n-x}Be(\alpha,\beta)}{\int\binom{n}{x}\theta(1-\theta)^{n-x}Be(\alpha,\beta)d\theta}=\frac{\Gamma(\alpha+\beta+n)}{\Gamma(\alpha+x)\Gamma(\beta+n-x)}\theta^{\alpha+x-1}(1-\theta)^{\beta+n-x-1}\propto Be(\alpha+x,\beta+n-x) \quad (4)$$

Under the condition that the total number of samples n and number of defects x are known, as long as the super parameter value of the prior distribution is figured out, the parameter of the posteriori distribution which is also the Beta distribution can be figured out according to formula (4), i.e. the shape parameter and size parameter of the reliability model of the tool library system can be obtained.

3 DETERMINATION OF HYPERPARA METER

It was assumed that $N_f(t)$ sets of equipment went wrong and $N_s(t)$ sets remained in good condition at the time point of t, and that $N_s(t+\Delta t)$ sets of equipment broke down and $N_s(t+\Delta t)$ sets of equipment remained in good condition at the time point of $t+\Delta t$, then the average number of equipment in good condition within the interval of Δt can be figured out. It is shown as follows:

$$\overline{N_s}(t)=\left[N_s(t)+N_s(t+\Delta t)\right]/2 \quad (5)$$

The average total working hours is $\overline{N_S}(t)g\Delta t$, and the number of sets of equipment that went wrong is:

$$N_f(t+\Delta t)-N_f(t)=\Delta N_f(t) \quad (6)$$

Then, the average failure rate within the time interval $(t,t+\Delta t)$ is as follows:

$$\bar{\lambda}(t)=\frac{N_f(t+\Delta t)-N_f(t)}{\overline{N}_s(t)\Delta t}=\frac{\Delta N_f(t)}{\overline{N}_s(t)\Delta t} \quad (7)$$

The average and variance of the failure rate of the prior information of the tool library system is:

$$\lambda=\frac{\bar{\lambda}_1(t),\bar{\lambda}_2(t)\cdots\cdots\bar{\lambda}_k(t)}{k} \quad (8)$$

and

$$S^2=\frac{1}{k}\sum\left[\bar{\lambda}(t)-\bar{\lambda}(t)\right]^2 \quad (9)$$

respectively.

The mean value and variance Of $Be(\alpha, \beta)$ distribution is:

$$\frac{\alpha}{\alpha+\beta}=\bar{\lambda} \quad (10)$$

$$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}=S_\lambda^2 \quad (11)$$

Let the average of the mean failure rate equal to the average of $Be(\alpha, \beta)$ distribution and the variance of the mean failure rate equals to that of $Be(\alpha, \beta)$ distribution (Huang, 2016; Kan, 2016). And then, the super parameters α, β can be figured out and serve as the prior distribution parameter of Bayesian estimation of the tool library system (Kaeding, 2015; Yang, 2015; Guo, 2017).

Eight failure time intervals of the tool library were selected, and the number of failures within the time intervals were figured out, which are shown in Table 2.

Table 2. Data Statistics of the Tool Library System Failure

Failure time interval	Failure number	Failure time interval	Failure number
0-50	1	200-300	1
50-100	1	300-400	0
100-150	1	400-500	1
150-200	3	500-600	2

The average failure rate $\overline{\lambda}(0\text{-}50)$ within the time interval of (0, 50) was figured out. Within the interval of (0, 50),

$\Delta t = 50$, $\Delta N_t(50) = 1$,

$N_s(0) = 55$, $N_s(50) = 54$,

$$\overline{N}_s(50) = \frac{1}{2}(55+54) = 54.5$$.

Based on formula (7), the average failure rate within the time interval was figured out:

$$\overline{\lambda}(0\text{-}50) = \frac{\Delta N_f(t)}{\overline{N}_s(t)\Delta t} = \frac{1}{5.45\times10^{-4}} / t$$

The rest can be done in the same manner. The average failure rates of the tool library failure data within other time intervals in Table 1 are figured out. The results are shown in Table 3.

Table 3. Average failure rate within each time interval of the tool library system

Failure time interval	Average failure rate	Failure time interval	Average failure rate
0-50	3.64E-04	200-300	1.83E-04
50-100	3.64E-04	3000-400	0.00E+00
100-150	3.64E-04	400-500	1.83E-04
150-200	1.12E-03	500-600	7.41E-04

The mean value and variance of the prior information of the tool library system can be figured out based on Formula (8) and (9).

$\lambda = 5.46\times10^{-4} / t, S^2 = 1.217\times10^{-7}$

Based on formula (10) and (11), the super parameter α, β of $Be(\alpha, \beta)$ distribution was obtained α=2.443, β=4476.530.

4 DETERMINATION OF POSTERIORI DISTRIBUTION

The estimation value of posteriori distribution of the tool library system can be obtained from Formula (4). For Weibull distribution, the average value of the failure rate within the total test time t_s is:

$$\hat{\lambda} = \frac{\int_0^{t_s} \lambda(t)}{t_s} = \frac{t_s^{m-1}}{\eta^m} \tag{12}$$

The variance of the failure rate is:

$$D(\lambda) = \frac{\int_0^{t_s} \left[\lambda(t) - \hat{\lambda}\right] dt}{t_s} = \frac{m^2}{\eta^{2m}} \cdot \frac{t_s^{2(m-1)}}{2m-1} - 2\hat{\lambda}\frac{t_s^{m-1}}{\eta^m} + \hat{\lambda}^2 \tag{13}$$

Based on Formula (4), the failure rate is subject to the $Be(\alpha+x, \beta+n-x)$ distribution, and the $Be(\alpha+x, \beta+n-x)$ distribution expectation is:

$$E(\lambda) = \frac{\alpha + x}{\alpha + \beta + n} \tag{14}$$

The variance of $Be(\alpha + x, \beta + n - x)$ distribution is as follows:

$$D(\lambda) = \frac{(\alpha+x)(\beta + n - x)}{(\alpha + \beta + n)^2(\alpha + \beta + n + 1)} \tag{15}$$

Eight sets of tool library system of this type were taken as the test sample of the reliability laboratory. 10 days of 24-hour reliability test was carried out on each test sample and 2 failure data were obtained.

Based on Formula (14) and (15), the mean value of the posteriori distribution of the tool library system is:

$E(\theta) = 9.90\times10^{-4}$

The variance of the posteriori distribution of the tool library system is:

$D(\theta) = 2.204\times10^{-7}$

According to Formula (12), (14), (13) and (15), the value of shape parameter and size parameter of

Weibull distribution of the tool library system were figured out, which is 0.70 and 1,868.784 respectively.

5 THE RELIABILITY MODEL OF THE TOOL LIBRARY SYSTEM

In view of the test in tool library laboratory, due to the impact of test expenditure and test duration, small samples are frequently seen. Therefore, this paper analyzes the reliability modeling technology for the small samples in the tool library laboratory. By applying the Bayesian theory, the probability density function $f(t)$, distribution function $F(t)$, reliability function $R(t)$ and failure rate function $\lambda(t)$ within the time be-tween failures of the tool library are obtained, and are shown as follows:

$$f(t)=\frac{0.70}{1868.784}\left(\frac{t}{1868.784}\right)^{-0.30}\exp\left[-\left(\frac{t}{1868.784}\right)^{0.70}\right] \quad (16)$$

$$F(t)=1-\exp\left[-\left(\frac{t}{1868.784}\right)^{0.70}\right] \quad (17)$$

$$R(t)=1-F(t)=\exp\left[-\left(\frac{t}{1868.784}\right)^{0.70}\right] \quad (18)$$

$$\lambda(t)=\frac{f(t)}{R(t)}=\frac{0.70}{1868.784}\left(\frac{t}{1868.784}\right)^{-0.30} \quad (19)$$

The graphs of probability density function $f(t)$, distribution function $F(t)$, reliability function $R(t)$ and failure rate function $\lambda(t)$ within the time between failures of the tool library are shown in Figure 1, 2, 3 and 4.

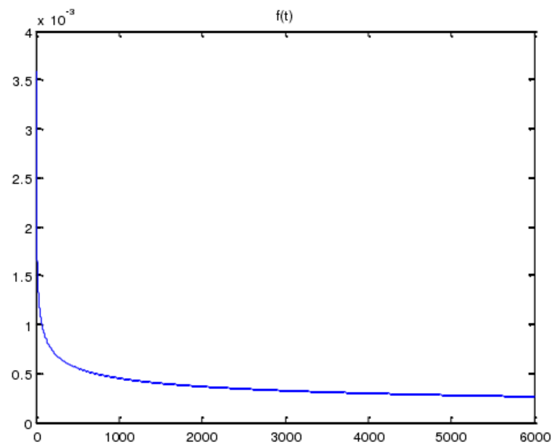

Figure 1. The probability density function graph of the probability density function

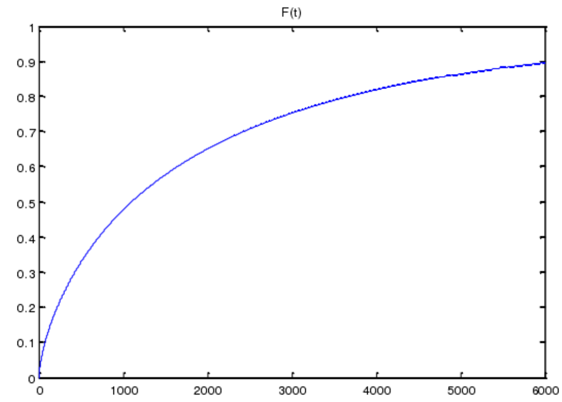

Figure 2. The distribution function graph of the tool library system

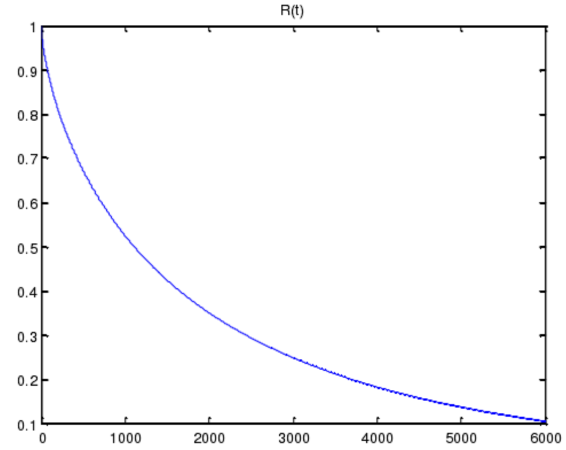

Figure 3. The reliability function graph of the tool library system

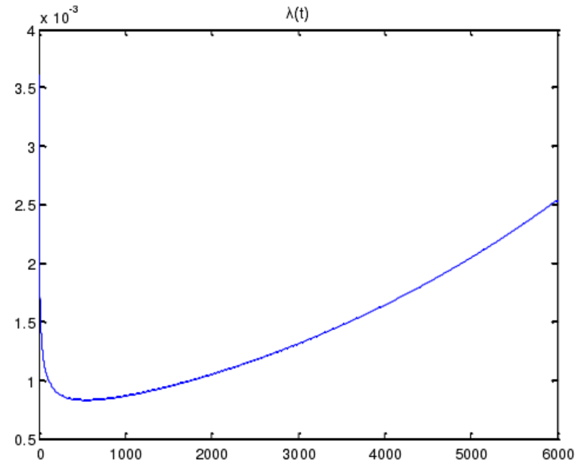

Figure 4. The failure rate function graph of the tool library system

6 CONCLUSIONS

Small sample database was obtained based on the test in the laboratory. Based on the Bayesian theory, and the failure rate at the early stage of data of the tool library system, this paper selects conjugate distribution to establish prior distribution and uses Generalized-Methods-of-Moments to estimate the prior distribution parameter. Finally, the test reliability model of the tool library system laboratory was obtained. The modeling results are consistent with the actual situation, indicating that this method is effective. This paper can provide theoretical basis to the design of reliability of the numerically-controlled machine tool.

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