DISCUSSION ON THE TRAJECTORY OPTIMIZATION OF MECHANICAL LINKAGE MECHANISM BASED ON QUANTUM GENETIC ALGORITHM

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ABSTRACT*: To remedy the shortcomings of genetic algorithm—more iteration and slow convergence speed, a new encoding quantum genetic algorithm based on quantum genetic algorithm (RQGA) was proposed. In carrying out the study with RQGA, the connecting rod that was widely used in mechanic engineering served as the study object; an optimization model of the problem was established by using real number encoding based on quantum genetic algorithm and used the algorithm to seek the optimization of solution. Detailed studied made in the application to the design of mechanical engineering using RQGA. At the same time, the results of genetic algorithm and basic quantum genetic algorithm were compared and analysed, which showed the feasibility and advantage of the proposed real coded quantum genetic algorithm. As for large deviations between the optimal solution and the target value using these algorithms—GA, QGA and RQGA, more efforts were needed to adjust the objective function to improve the algorithm's ability in solving mechanical optimization problems.*

KEYWORDS: *genetic algorithm; quantum computation; linkage mechanism; trajectory optimization*

1 INTRODUCTION

Mechanical optimization design is borrow and application of optimization design in the field of machinery. Its basic idea is: first, analyse the engineering design problem as analysing an optimization problem, and then select the appropriate optimization method that meets the demands of solving the problem through computer (Xiang et al., 2016; Eck et al., 2015). As the basis and core of the optimization design, much attention has been paid to optimization algorithm by the engineers and scientists. It has also been greatly developed. Genetic Algorithm (GA) is a more commonly used modern design method. It has been widely applied for its good robustness and wide adaptability (Jamshidi et al., 2015; Bi et al., 2015).

However, the genetic algorithm also has many shortcomings, such as much iteration, slow convergence speed, and is liable to fall into the local extremum. Along with the development of quantum computing, the development of the concept of qubits and quantum superposition, Quantum Genetic Algorithm (QGA) came into being. It is used to optimize the search capability with a combination of the quantum computing theory and genetic algorithm method. Practice has proved that the quantum genetic algorithm is better than genetic algorithm in terms of the diversity of the population, solution stability and speed of solving a problem RQGA (Liu et al., 2015; Zheng et al., 2016; Chen et al., 2016). A new type of GA is proposed in the paper: Real Coded Quantum Genetic Algorithm (RQGA) and it is used to solve the optimization problem in typical mechanical linkage. Through the comparison with GA and QGA analysis of the optimization results, it is confirmed that RQGA is viable and has an advantage over other algorithms. This helped promote the further development of quantum genetic algorithm.

2 METHODOLOGY

2.1 Mechanical optimization design

The definition of mechanical optimization design: in a given work load or working conditions; also fits in the design limit of mechanical products, product shape, geometric relations or other factors of mechanical products. The mechanical function, strength, economic efficiency and other properties of the system are set as the optimization object. Then, establishing appropriate design variables, constraint conditions and objective functions. All these efforts are done for the objective function to get the optimal solution. Mechanical optimization design has important significance in improving the quality of products, shortening production cycle and reducing the cost. It is widely used in the integrated design, mechanism design, special machinery and general machinery parts design and process design, and certain results are achieved (Baykasoğlu et al.,2015; Ran et al.,2015).

The mathematical model of the optimal design problem is the mathematical abstraction of the actual optimization problem. Assume there are n design variable (the count of n starts from 1), X= [x1, X2...Xn] T. Well satisfying the inequality constraints $g_u(X)=g_u(x_1,x_2,\ldots,x_n)\geq 0(u=1,2,\ldots,m)$ and equality constraints $f_v(X)=f_v(x_1,x_2,\ldots,x_n)=0$ （v=1,2,…,p<n） , get minimum of the target function $f(X)=f(x_1,x_2,\ldots,x_n)$. This optimization problem can be abstracted as mathematical model, in mathematical expression:

$$\begin{cases} \min : f(X) \\ s.t.: \begin{array}{l} g_u(X) \geq 0 \ (u=1,2,\text{L} \ ,m) \\ h_v(X) = 0 \ (v=1,2,\text{L} \ ,p<n) \end{array} \end{cases} \quad (1)$$

In mechanical design, optimization can be used in many fields, and the type and nature of the problem also varies. For the problem of function optimization, the common mechanical optimization problem is shown in Figure 1 as follows:

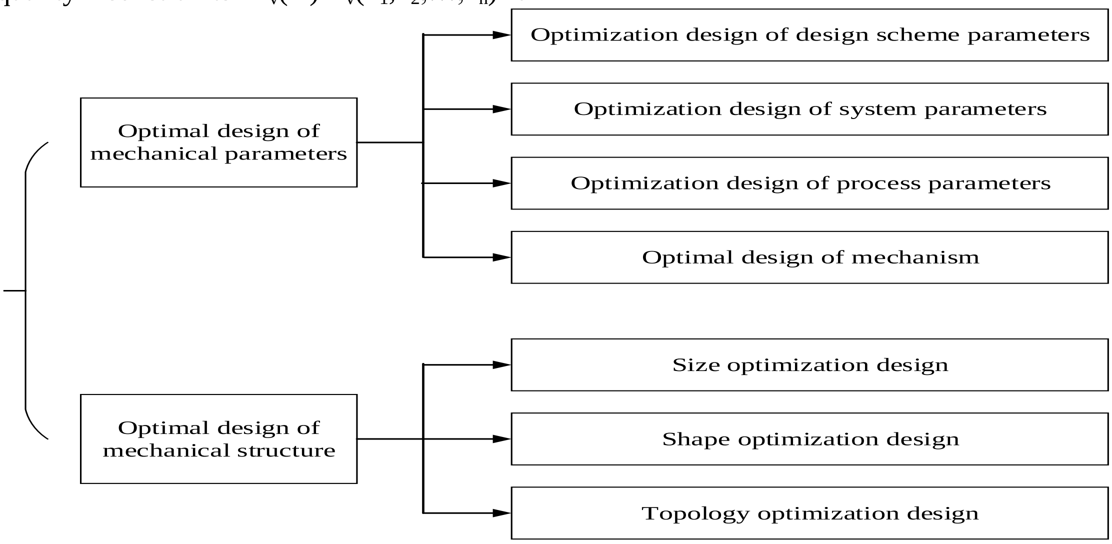

Figure 1. Common types of mechanical optimization problems

At present, the general optimization of mechanical optimization is mainly for the optimization of structural parameters. The whole process of mechanical optimization design in general can be divided into the following steps: engineering design specific object is abstracted as a mathematical model of optimal design; select the appropriate optimization methods according to different problems; write computer program; prepare necessary initial data, program debugging and optimize computing process; analyse the calculation results, if the results of its qualitative analysis and conventional design has too big a gap is or not consistent with regular results, then the rationality of the mathematical model should be re-examined. The schematic diagram of the optimization process is shown in Figure 2.

2.2 Quantum genetic algorithm

Quantum genetic algorithm (QGA) is a probability optimization method based on the principle of quantum computing. The characteristics of the algorithm are: small population size, fast convergence speed and strong global search ability, which make up for the premature convergence and potential to fall into the local extreme value of genetic algorithm. Therefore, quantum genetic algorithm has been widely applied in many fields(Dey et al., 2014).

In a quantum computer, the smallest unit of information is represented as the qubit, also known as quantum bits, a qubit can not only represent the two states, 0 and 1, but also represents any superposition between the two states. A qubit may be in the state $|0\rangle$, or $|1\rangle$, or the superposition between the two, so a qubit state can be expressed as:

$$|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (2)$$

Among them, α and β are two complex constants, which represent the probability of the 0 and 1 states of qubits, and fit the normalization conditions:

$$|\alpha|^2 + |\beta|^2 = 1$$

(3)

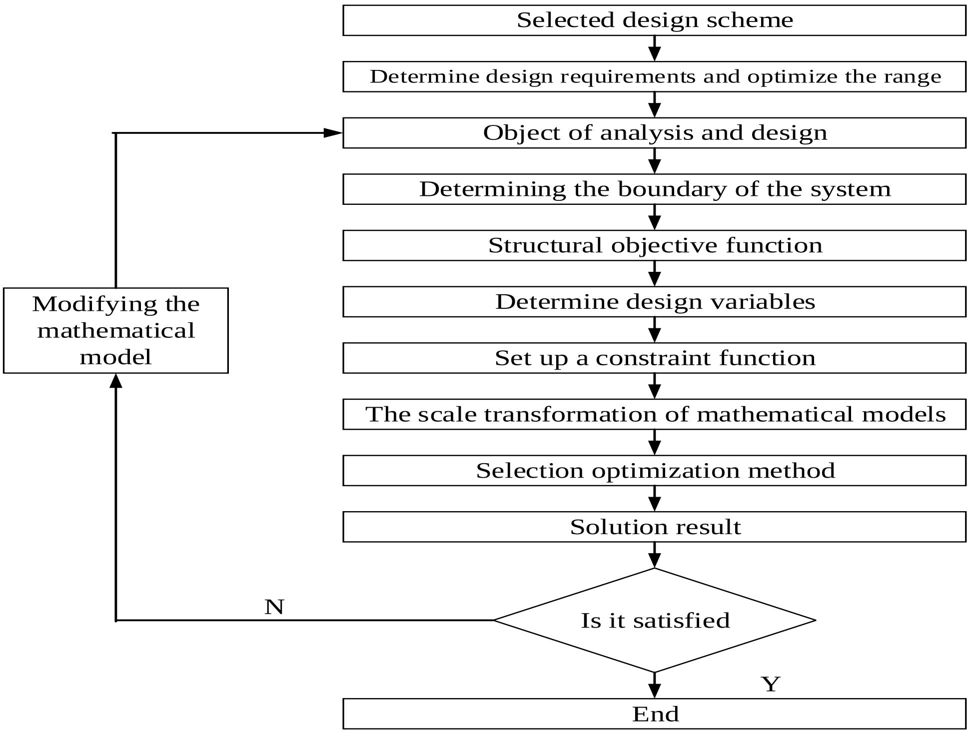

Figure 2. Optimization process diagram

$|\alpha|^2$ and $|\beta|^2$ are the probability for quantum bits in 0 state and 1 state respectively. It is known from the above expression that a qubit is a probability expression of 0 state and 1 state and the state can be "0", "1" or an arbitrary linear superposition of the state of the two. By observing it, it will collapse into a determined single state.

Traditional genetic algorithm adopts crossover and mutation operations to maintain population diversity. Quantum genetic algorithm realizes the requirement of group diversity by using quantum logic gates acting on the probability amplitude of quantum states (SaiToh et al., 2014). Methods using binary bits, the fitness value and the probability amplitude comparison method to update the quantum gates are only for combinatorial optimization problem with a known optimal solution. For actual optimization problem with no knowledge of its optimal solution principle, updated chromosomes by quantum rotation gates, the specific operation is as follows:

$$\begin{bmatrix} \alpha_i^{t+1} \\ \beta_i^{t+1} \end{bmatrix} = \begin{bmatrix} \cos(\theta) - \sin(\theta) \\ \sin(\theta) - \cos(\theta) \end{bmatrix} \cdot \begin{bmatrix} \alpha_i^t \\ \beta_i^t \end{bmatrix} \qquad (4)$$

In the formula, θ is the rotation angle, and according to the rotation strategy in Table 1, take: $\theta = s(\alpha_i,\beta_i)\cdot\Delta\theta$. In the table, x and b are respectively the optimal solution of the current solution and the generation, x_i and b_i are respectively corresponding to the number i gene in the binary chromosome, f (x) and f (b) respectively correspond to the fitness value, when $f(x) \geq f(b)$, solution x is superior to b solution.

Table 1. Rotation angle selection strategy

x_i	b_i	$f(x)\geq f(b)$	$\Delta\theta$	$s(\alpha_i,\beta_i)$			
				$\alpha_i\beta_i>0$	$\alpha_i\beta_i<0$	$\alpha_i=0$	$\beta_i=0$
0	0	false	0	0	0	0	0
0	0	true	0	0	0	0	0
0	1	false	0	0	0	0	0
0	1	true	0.05π	-1	+1	±1	0
1	0	false	0.01π	-1	+1	±1	0
1	0	true	0.025π	+1	-1	0	±1
1	1	false	0.005π	+1	-1	0	±1
1	1	true	0.025π	+1	-1	0	±1

The realization of the basic genetic algorithm includes determining encoding scheme; generating initial population; set fitness function; selection of control parameters, including population size, maximum iteration, the probability of executing genetic operation and some other auxiliary parameters; to determine the genetic operators and selection strategies, including reproduction, crossover and mutation and other advanced operation; set the stopping criterion for (Asadi et al., 2014). The general process of a quantum genetic algorithm is similar to that of genetic algorithm, as illustrated in Figure 3.

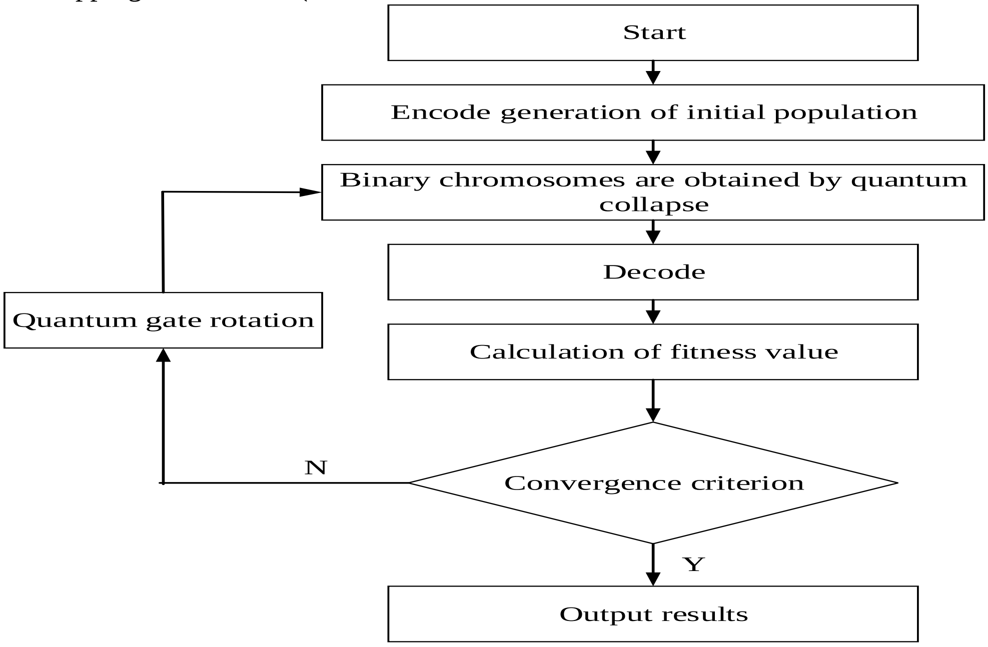

Figure 3. Quantum Genetic Algorithm Flow

2.3 Linkage mechanism optimization

Linkage mechanism is a common mechanism in the mechanical engineering, which is mainly used for the transmission of the mode of motion (Pellicciari et al., 2015) as shown in Figure 4. The traditional mechanism design methods include graphic method, drawing method and experimental method. However, the design accuracy and design efficiency of these methods cannot meet the requirements of modern mechanical transmission. The basic problem of Linkage mechanism design is to, under the requirements of certain structural conditions, dynamic conditions and continuity of motion, select the type of the mechanism according to the given requirements, and determine the size of each part.

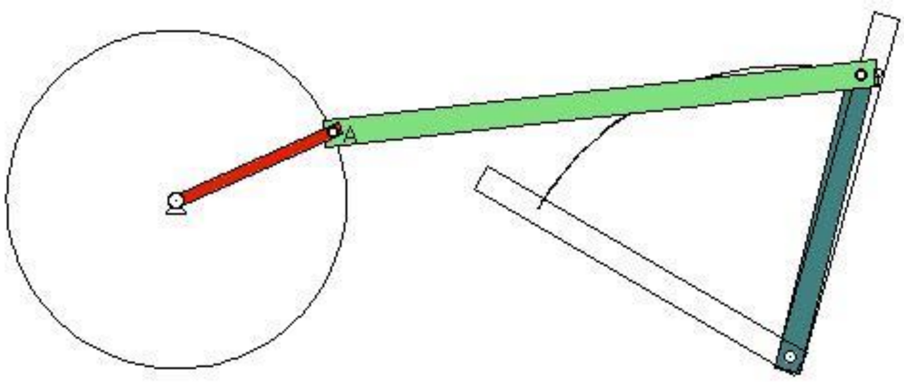

Figure 4. Multi-link institutions

According to the known conditions from the mechanism, the parameters of the kinematic diagram of the mechanism are the main problem of the design of the planar linkage mechanism (Tsuge et al., 2016). The parameters of the kinematic diagram include: the distance between centres of the rotation pair, the location of the mobile pair, and the location and size of the point that describes the connecting rod curve. Known condition is set according to the use of the mechanism, so it is in various form, accordingly, the actual problems of linkage mechanism are of various type. But these problems can be summarized as two kinds of basic problems: implementation of known motion rules, namely, when the motion law of the driving component are known, requirements are to ensure the implementation parts move according to the motion given; realize the known trajectory, which is to ensure that parameters determined by the kinematic diagram of mechanism can see that any point in the complex plane motion of the mechanism moves along a given trajectory motion.

In this paper, a crank rocker mechanism is designed to realize the trajectory of M points on the connecting rod. The given coordinates of point M are shown as shown in Table 2.

Table 2. M point trajectory requirements

	1	2	3	4	5	6	7	8
XM_i	25	24	21	16	14	11	20	30
YM_i	15	17	17	15	16	12	8	13
$\Delta\theta_i$	0°	22°	44°	66°	88°	130°	220°	314°

Set up the coordinate system as shown below, the coordinates of the four bar mechanism in a certain time point M is expressed as [Mx, My], which is determined by the coordinates of point A, [Ax, Ay]. L1, L2, L3, L4, L5 each is the length of each rod. B and δ are the incidence angles of the rod and rack respectively, and the intersection angle of the crank is θ. Assume that the auxiliary angle γ, ε, η, are, as shown in figure 5 t:

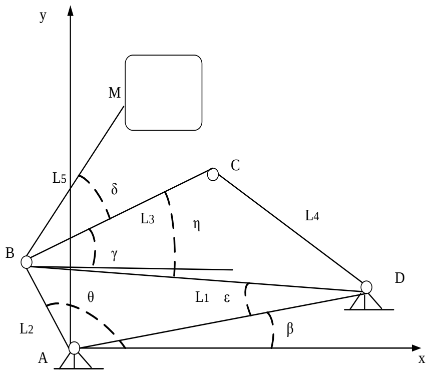

Figure 5. Four-bar mechanism

It can be inferred from the above geometrical relationship

$$\begin{aligned} Mx &= Ax + L_2\cos\theta + L_5\cos(\gamma+\delta) \\ My &= Ax + L_2\sin\theta + L_5\sin(\gamma+\delta) \end{aligned} \tag{5}$$

$$\eta = \cos^{-1}\frac{L_1^{\,2}+L_2^{\,2}+L_3^{\,2}-L_4^{\,2}-2L_1L_2\cos(\theta-\beta)}{2L_3\left[L_1^{\,2}+L_2^{\,2}-2L_1L_2\cos(\theta-\beta)\right]^{1/2}} \tag{6}$$

$$\varepsilon = \tan^{-1}\frac{L_2\sin(\theta-\beta)}{L_1-L_2\cos(\theta-\beta)} \tag{7}$$

$$\gamma = \eta-(\varepsilon-\beta) \tag{8}$$

The coordinates of the point M can be calculated at A point coordinates, the length of each rod, B and δ, the incidence angles of the rod and rack respectively, and the intersection angle of the crank θ. When the value of θ is taken continuously, the coordinate locus of the M point is the predetermined path.

3 RESULTS AND DISCUSSION

3.1 Real coded quantum genetic algorithm and its implementation

The core of the real coded quantum genetic algorithm is to code the qubits with real numbers. In the real coded quantum genetic algorithm, the encoding method uses real number coding, which can omit the encoding and decoding process and has a high precision of calculation. The specific steps are shown in Figure 6.

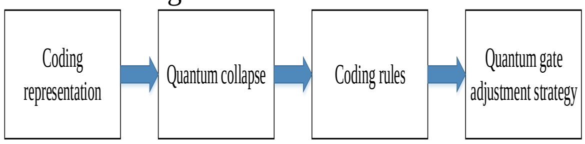

Figure 6. The steps of real coded quantum genetic algorithm

The computer technology applied to the quantum genetic algorithm in this paper is Java language, and the programming environment is JBuilder 2005. Based on the features of the Java language and the JBuilder, the system structure diagram of the visual operating system developed in this paper is shown in Figure 7 below. The basic idea of developing the system is: first runs the program, then pops up the visual interface; the optimization problem and optimization algorithm are selected on the visual interface, and the necessary control parameters are put in at the same time, then carry out the optimization, and finally, the optimal solution is generated from the visual interface.

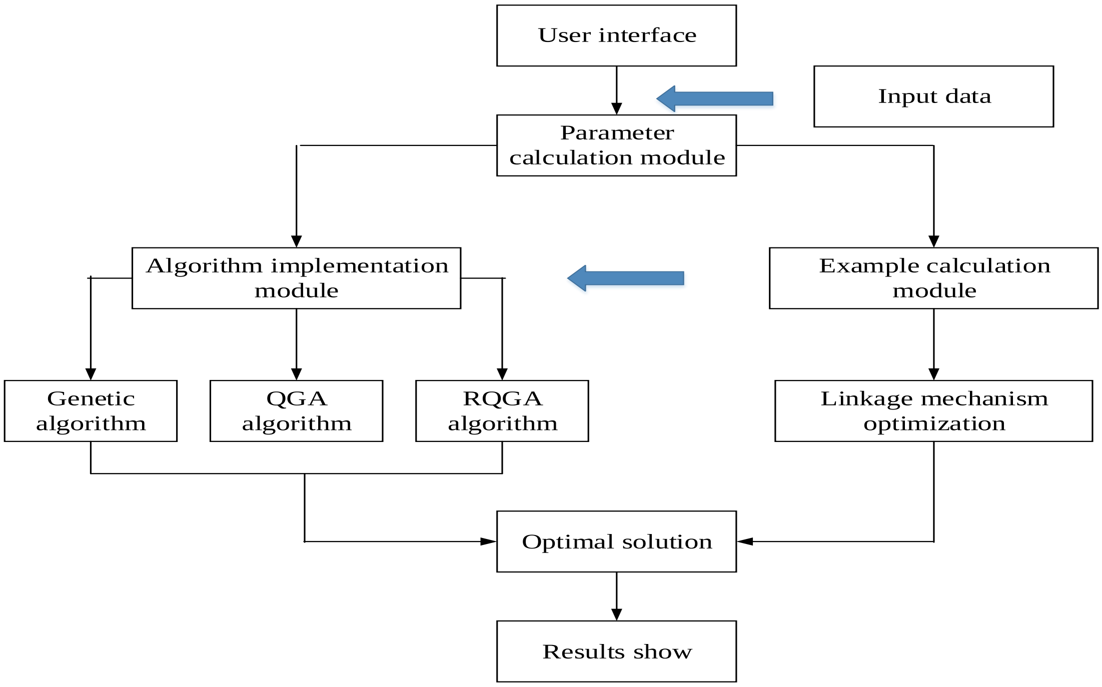

Figure 7. System structure diagram

3.2 The establishment of an optimal model

The key problem to be solved in mechanical optimization design is how to transform the actual mechanical design problem into a mathematical model that is easy to solve. Whether the mathematical model can reflect the engineering practical problems accurately and concisely determines if the optimization results are useful. The basic elements of a mathematical model are shown in Figure 8.

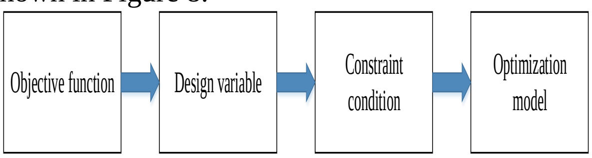

Figure 8. Basic elements of the model

The establishment of the objective function is a key step in the whole optimization design, and the establishment of the objective function must meet the requirements of the design criteria. Generally speaking, in the optimization design of the mechanism the objective function is expressed by motion error and dynamic characteristics, and in the optimization design of parts the objective function is expressed in terms of mass, volume, efficiency, reliability and carrying capacity.

In the optimization problem, the design variables must be independent. In general, the more the design variables are, the greater the amount of calculation is. Therefore, for a mechanical optimization problem, the number of design variables must be selected properly. The basic principle is to minimize the number of design variables, that is to say, as far as possible, those parameters that have little influence on the objective function are taken as the given parameters, and the parameters that affect the objective function immensely are taken as the design variables, so as to simplify the mathematical model of optimization design.

Any practical problem of mechanical engineering is to find the optimal solution under certain conditions. These conditions are called constraints in the optimization design. The constraints can be divided into two categories, performance constraint and boundary constraint. Any kind of constraints must meet the following principles when it is determined as the constraint conditions: no contradictory constraints; minimize unnecessary constraints; not to leave out the necessary constraints, give the upper and lower constraints as possible.

In this paper, the example of optimization is constrained optimization problem, while SGA, QGA and RQGA are only suitable for the unconstrained optimization problem solving, so we must adopt some methods for constrained optimization problem to transform into unconstrained optimization problem, and then the start optimization. The premises are: cannot damage constraints condition of the constraint problem; constrained optimization problems are attributed to the same optimal solution of the constrained problem is transformed to. Therefore, the internal point penalty function method is adopted, and the optimization model is as follows:

$$\begin{cases} \min : \Phi(x,r) = f(x) + r\sum_{j=1}^{7}\dfrac{1}{g_j(x)} \\ g_1(X) = L_1 - L_2 \ge 0 \\ g_2(X) = L_3 - L_2 \ge 0 \\ g_3(X) = L_4 - L_2 \ge 0 \\ s.t.: g_4(X) = L_1 + L_3 - L_2 - L_4 \ge 0 \\ g_5(X) = L_4 + L_3 - L_2 - L_1 \ge 0 \\ g_6(X) = L_1 + L_4 - L_2 - L_3 \ge 0 \\ g_7(X) = \delta\min - 40^0 \ge 0 \end{cases} \quad (9)$$

3.3 Analysis of optimal solution results

Find solutions to the mechanical optimization problems by SGA, QGA and RQGA respectively. The results are analysed and compared.

The standard genetic algorithm adopts binary encoding, roulette selection algorithm, single point crossover and gene 0-1 mutation. The population size M=10, crossover probability p_c=0.5 and mutation probability p_m=0.01 are adopted. The best solution reservation strategy is adopted, and the maximum iteration number T=500 is set as the convergence condition. Quantum genetic algorithm and real coded quantum genetic algorithm generate initial coding randomly, use the best solution preserving mechanism to update chromosomes, and use maximum iteration number T=500 as convergence condition, and the population size is M=10. The program is programmed by JAVA language. The standard genetic algorithm, quantum genetic algorithm and real coded quantum genetic algorithm are used for 20 times each in obtaining optimization solutions. The distribution of the optimal solutions of each of the three optimization algorithms is shown in Figure 9.

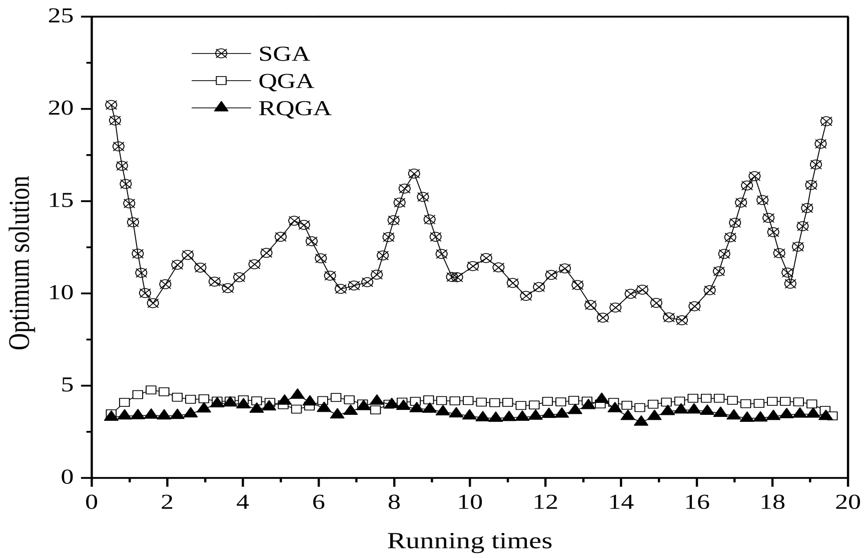

Figure 9. Optimal solution distribution graph

As shown in Figure 9, after the end of the operation, the optimal solution obtained by the three optimization algorithms is shown in Table 3, as shown in Table 3.

Table 3. The optimal solution obtained by each algorithm

Paramet er	A x	A y	L 1	L 2	L 3	L 4	L 5	β	δ	θ_1	F (X)
SGA	1 1.1	0. 0	3 2.1	8. 8	3 0.0	2 2.6	1 3.3	8. 9	3. 0	1. 0	8. 19
QGA	1 2.0	3. 9	5 8.9	7. 9	5 8.1	1 1.5	9. 9	- 16.8	6. 8	6 0.0	3. 01
RQGA	1 3.9	3. 0	5 2.8	8. 0	4 9.9	1 8.4	9. 8	- 11.9	1 3.8	4 9.9	3. 59

Collect the statistics of the operation results, run it 20 for times. The statistical results of the study are as follows: the optimal value, the optimal QGA is better than RQGA's solution, and is better than the SGA optimal solution of RQGA, namely, quantum genetic algorithm has strong searching ability; average value, run it 20 for times, the average income is better than that of RQGA optimal QGA value, the average value is better than that of the SGA optimal solutions of RQGA, namely the optimal quantum genetic algorithm solution the average value is better; the standard deviation, run it

20 times, optimal solutions of RQGA standard deviation is better than that of QGA, the optimal QGA's solution of the standard deviation is better than SGA, which is the stability of RQGA is better than that of QGA solution and stability of QGA is better than of SGA. Comparing the average running time and run it for 20 times. Then we got an average running speed (15.6s) of RQGA and that of SGA (45.7s), and the average running speed of SGA is better than that of QGA (68.1s), which means that the quantum genetic algorithm has faster solution speed than that of SGA (68.1s).

Because this example is trajectory optimization problem, we can calculate the trajectories obtained by the optimal solution of all optimization algorithms, and the distribution of each location point illustrated in the graphics is shown in Figure 10 and Figure 11.

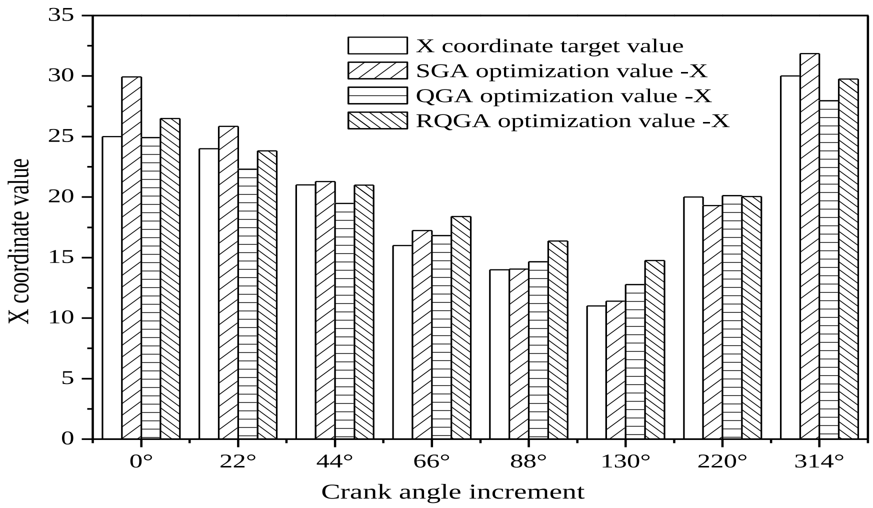

Figure 10. X coordinate value diagram

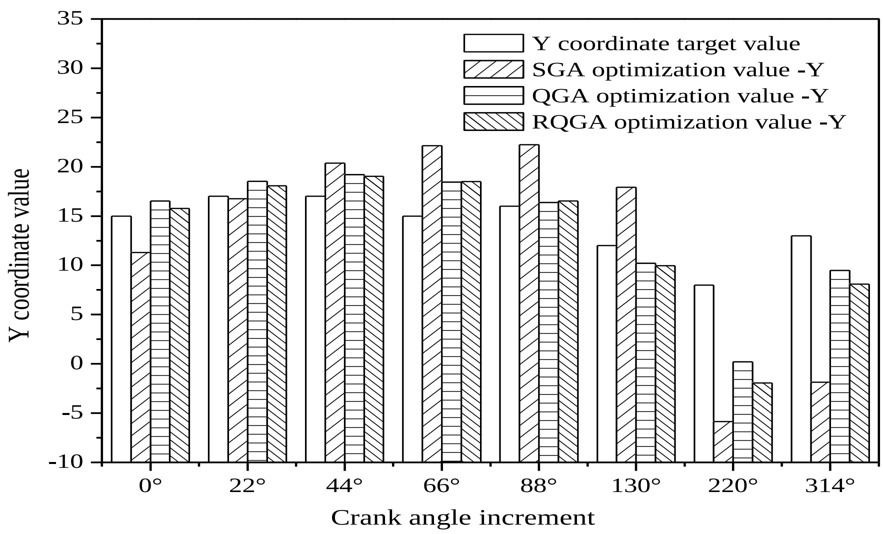

Figure 11. Y coordinate value diagram

According to the chart, although the quantum genetic algorithm can get better results than the genetic algorithm, but in this case, there was a large deviation between the target coordinates and coordinates obtained in three kinds of optimization methods. The cause of this problem may be: the selection of the objective function is not reasonable. Because the objective function is the standard deviation between the actual calculated value of each point and the target value, it may be the case that the best solution is obtained when some point deviations are small and some point deviations are large.

3.4 Model improvement and scheme comparison

The optimization results show: compared with SGA, the encoding and decoding process are reduced so the solution speed is improved since RQGA adopts real number encoding. Meanwhile, RQGA uses quantum gates to update chromosomes to enhance its optimization performance. However, it is obvious that when F1 (X) is used as the objective function, there are large deviations between the optimal solution and the target value. There are two reasons for the excessive deviation. One is the unreasonable selection of the objective function, and the other is that the optimization algorithm is not suitable. Therefore, we improve the objective function and investigate the factors that cause large deviations.

The linear weighting method F2 (X) is used to optimize this example and operates 20 times. The optimal solution distribution of the three optimization algorithms is shown in Figure 12 as shown in Figure 12.

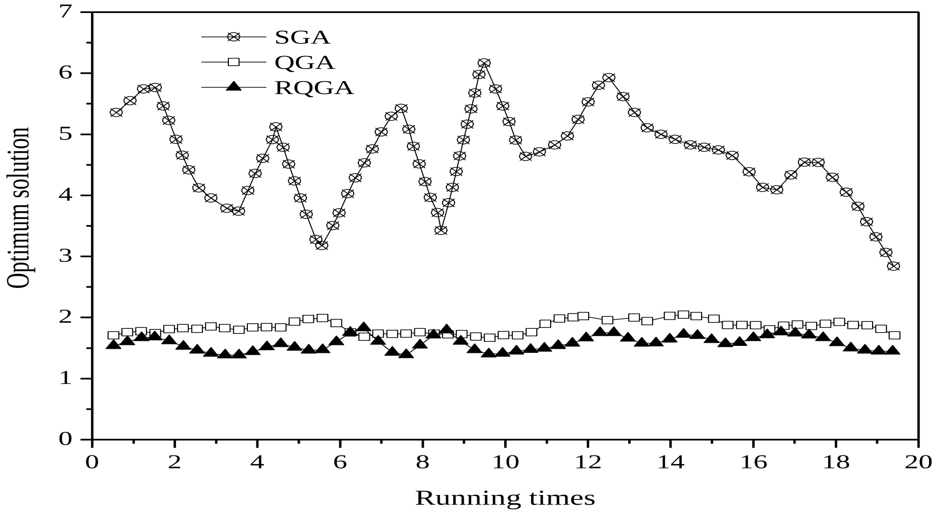

Figure 12. Optimal solution distribution chart based on $F_2(X)$

The analysis of examples shows that the optimization results in each position must be as close as possible to the target value, so deviation of each position can be used as the objective function F3 (X), run the algorithm 20 times to see the optimal solution for three kinds of optimization algorithm. The distribution chart is shown in figure 13:

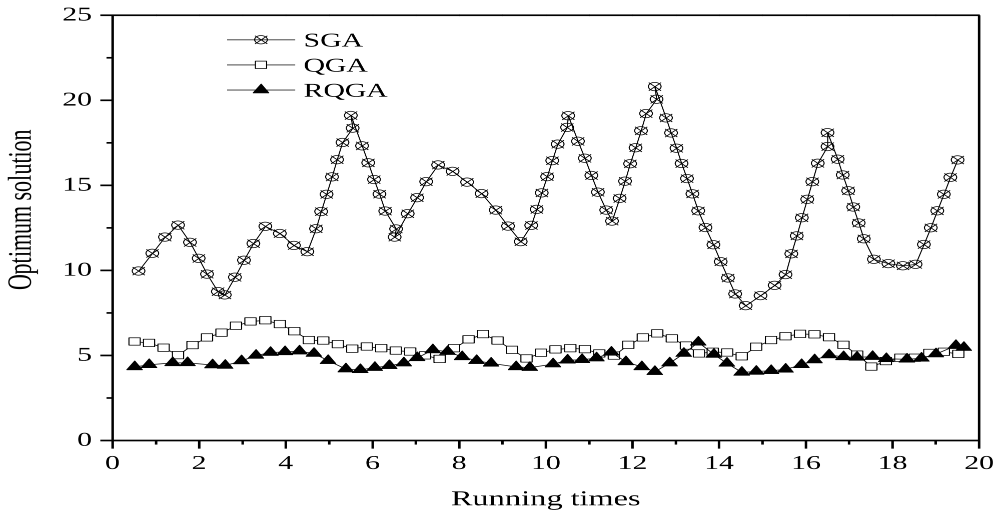

Figure 13. Optimal solution distribution map based on $F_3(X)$

After running 20 times, the optimal solution of QGA is better than RQGA, and the optimal solution obtained by RQGA is better than RQGA. This optimization model confirms the conclusion that the optimization ability of quantum genetic algorithm is better than that of standard genetic algorithm. The optimization ability of quantum genetic algorithm is stronger. After using real number coding, its solution stability and result producing speed have been improved.

Because the aim of the optimization examples is for the coordinates of the optimal solution to close to the value of the target value maximumly, so the evaluation index of all measures can only be the deviation of each point. The maximum deviation produced from these three kinds of objective functions are as shown follows:

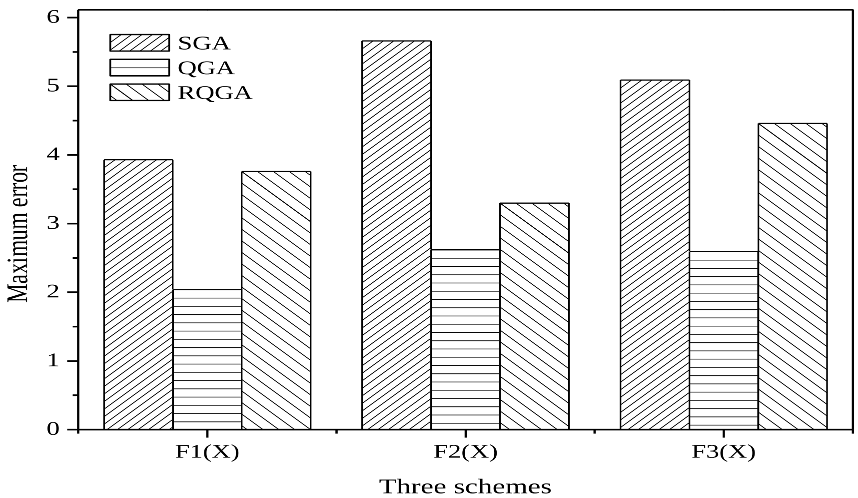

Figure 14. The maximum deviation of the three schemes

The data in Figure 14 shows: because F3 (X) uses error as the objective function, making the optimization result rather intuitive, the data in the table shows that by using SGA, QGA and RQGA three kinds of algorithm to optimize the examples illustrated in this chapter, the deviation obtained by optimization result is rather large; to the same objective function, RQGA has better solution stability and faster calculation speed. Therefore, on the one hand, the optimization results of this example prove that the quantum genetic algorithm has better performance in searching, stability and solution speed. On the other hand, for this instance, the optimization algorithm adopted in this paper is not satisfactory.

4 CONCLUSION

In this paper, a new real number encoded quantum genetic algorithm (QGA) is proposed based on the quantum genetic algorithm. The trajectory of planar four bar is optimized by real encoding quantum genetic algorithm. The optimization results are compared with that of genetic algorithm and quantum genetic algorithm. By comparing average value, the optimal value, standard deviation and average operation time of the four parameters, it shows that the speed of finding solutions is faster and more stable under real number encoding quantum genetic algorithm in the optimization of the multi variable mechanism. As to the linkage path optimization problem, there are large deviations between the target value and coordinate optimal solutions of the three algorithms. To remove this roadblock, the original objective function was improved, after optimization, it is found that the deviation between the produced coordinate value and the target value of is still large.

From the optimized object, the planar four bar mechanism is only an individual case. In the case of four bar linkage trajectory optimization, the deviation from all algorithms is large, and the algorithm itself can be improved from two aspects: coding scheme and quantum gate adjustment. Meanwhile, the algorithm needs to be verified and improved in practical multi-parameter engineering optimization, in order to increase the application scope of the algorithm and improve the algorithm's capability to solve mechanical optimization problems.

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