THE KINEMATIC MODEL OF 5R ARTICULATED INDUSTRIAL ROBOT USED IN WELDING PROCESSES

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ABSTRACT: The paper presents the determination of the kinematic model equations for the 5R-type articulated industrial robot that can be applied in welding processes. The method used to perform this modeling is the iterative method and it takes the advantages of the previously determined equations of the geometric model. The operational kinematic parameters (linear and angular velocities and accelerations) are eventually determined. They represent the equations of the direct kinematic model.

KEY WORDS: kinematic model, articulated robot, welding.

1 INTRODUCTION

An important step in modeling a robot is the determination of kinematic model equations. They express the operational kinematic parameters (linear and angular velocities and accelerations), based on the geometric parameters (such as the constructive dimensions and generalized coordinates) and the kinematic generalized coordinates (velocities and accelerations from the robot's joint). The iterative method, described in (Negrean et al., 2008), (Ispas, 2003) and (Deteșan, 2015) is used at this step of kinematic modelling.

2 INITIAL DATA

After the geometric modeling of the 5R articulated robot was performed (Bugnar et al., 2013), the data saved in the file *RRRRR_geo.mat* will be loaded, and it will represent a part of the initial data in the kinematic computation.

The iterative method will be implemented into the script file *RRRRR_kin.m*, as presented in (Detesan, 2015), (Gui et al., 2014), (Horvat et al., 2010), (Detesan, 2008), using the symbolic computation in MATLAB (Detesan & Ispas, 2009).

The joint axes versors, requested into the modeling equations, are defined as:

$$\left[\bar{k}\right]_1^1=\begin{bmatrix}0\\0\\1\end{bmatrix};\quad \left[\bar{i}\right]_2^2=\begin{bmatrix}1\\0\\0\end{bmatrix};\quad \left[\bar{i}\right]_3^3=\begin{bmatrix}1\\0\\0\end{bmatrix};\quad (1)$$

$$\left[\bar{j}\right]_4^4=\begin{bmatrix}0\\1\\0\end{bmatrix};\quad \left[\bar{i}\right]_5^5=\begin{bmatrix}1\\0\\0\end{bmatrix};\quad \left[\bar{i}\right]_6^6=\begin{bmatrix}1\\0\\0\end{bmatrix}.\quad (2)$$

The kinematic elements of the robot's base are given by the equations:

$$\left[\bar{\omega}\right]_0^0=\begin{bmatrix}0\\0\\0\end{bmatrix};\ \left[\bar{v}\right]_0^0=\begin{bmatrix}0\\0\\0\end{bmatrix};\ \left[\bar{\varepsilon}\right]_0^0=\begin{bmatrix}0\\0\\0\end{bmatrix};\ \left[\bar{a}\right]_0^0=\begin{bmatrix}0\\0\\g\end{bmatrix}.\quad (3)$$

From the homogeneous transformation matrices, as a result of the geometric model, the following rotation matrices between adjacent frames are determined:

$$\left[R\right]_1^0=\begin{bmatrix}cq_1 & -sq_1 & 0\\ sq_1 & cq_1 & 0\\ 0 & 0 & 1\end{bmatrix};\quad (4)$$

$$\left[R\right]_2^1=\begin{bmatrix}1 & 0 & 0\\ 0 & cq_2 & -sq_2\\ 0 & sq_2 & cq_2\end{bmatrix};\quad (5)$$

$$\left[R\right]_3^2=\begin{bmatrix}1 & 0 & 0\\ 0 & cq_3 & -sq_3\\ 0 & sq_3 & cq_3\end{bmatrix};\quad (6)$$

$$\left[R\right]_4^3=\begin{bmatrix}cq_4 & 0 & sq_4\\ 0 & 1 & 0\\ -sq_4 & 0 & cq_4\end{bmatrix};\quad (7)$$

$$\left[R\right]_5^4=\begin{bmatrix}1 & 0 & 0\\ 0 & cq_5 & -sq_5\\ 0 & sq_5 & cq_5\end{bmatrix};\left[R\right]_6^5=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix},\quad (8)$$

and also with respect to the fixed frame:

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$$[R]_5^0 = \begin{bmatrix} cq_1cq_4 - s(q_2+q_3)\cdot sq_1sq_4 & sq_5cq_1sq_4+s(q_2+q_3)\cdot cq_4sq_1 - c(q_2+q_3)\cdot cq_5sq_1 & cq_5(cq_1sq_4+s(q_2+q_3)\cdot cq_4sq_1)+ c(q_2+q_3)\cdot sq_1sq_5 \\ cq_4sq_1+ s(q_2+q_3)\cdot cq_1sq_4 & sq_5(sq_1sq_4 - s(q_2+q_3)\cdot cq_1cq_4) + c(q_2+q_3)\cdot cq_1cq_5 & cq_5(sq_1sq_4 - s(q_2+q_3)\cdot cq_1cq_4) - c(q_2+q_3)\cdot cq_1sq_5) \\ -c(q_2+q_3)\cdot sq_4 & s(q_2+q_3)\cdot cq_5 + c(q_2+q_3)\cdot cq_4sq_5 & c(q_2+q_3)\cdot cq_4cq_5 - s(q_2+q_3)\cdot sq_5 \end{bmatrix}; \quad (9)$$

$$[R]_6^0 = \begin{bmatrix} cq_1cq_4 - s(q_2+q_3)\cdot sq_1sq_4 & sq_5cq_1sq_4+s(q_2+q_3)\cdot cq_4sq_1 - c(q_2+q_3)\cdot cq_5sq_1 & cq_5(cq_1sq_4+s(q_2+q_3)\cdot cq_4sq_1)+ c(q_2+q_3)\cdot sq_1sq_5 \\ cq_4sq_1+ s(q_2+q_3)\cdot cq_1sq_4 & sq_5(sq_1sq_4 - s(q_2+q_3)\cdot cq_1cq_4) + c(q_2+q_3)\cdot cq_1cq_5 & cq_5(sq_1sq_4 - s(q_2+q_3)\cdot cq_1cq_4) - c(q_2+q_3)\cdot cq_1sq_5) \\ -c(q_2+q_3)\cdot sq_4 & s(q_2+q_3)\cdot cq_5 + c(q_2+q_3)\cdot cq_4sq_5 & c(q_2+q_3)\cdot cq_4cq_5 - s(q_2+q_3)\cdot sq_5 \end{bmatrix}. \quad (10)$$

From the same transformation matrices, the following position vectors between successive frames will be extracted:

$$\bar{r}_1^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \quad \bar{r}_2^1 = \begin{bmatrix} 0 \\ l_1 \\ l_2 \end{bmatrix}; \quad \bar{r}_3^2 = \begin{bmatrix} 0 \\ l_3 \\ 0 \end{bmatrix}; \quad (11)$$

$$\bar{r}_4^3 = \begin{bmatrix} 0 \\ l_4 \\ 0 \end{bmatrix}; \quad \bar{r}_5^4 = \begin{bmatrix} l_6 \\ l_5 \\ 0 \end{bmatrix}; \quad \bar{r}_6^5 = \begin{bmatrix} l_7 \\ 0 \\ 0 \end{bmatrix}. \quad (12)$$

The parameters from (1)-(12) represent the initial data required into the equations of the kinematic iterative method, as it will be obvious in the following steps.

3 THE KINEMATIC MODEL

3.1 The Inverses of the Rotation Matrices

Considering the fact that the inverse of a rotation matrix equals the transpose matrix, the inverses of the rotation matrices between successive frames from the equations (4)-(8) will be determined as:

$$[R]_0^1 = \begin{bmatrix} cq_1 & sq_1 & 0 \\ -sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad (13)$$

$$[R]_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_2 & sq_2 \\ 0 & -sq_2 & cq_2 \end{bmatrix}; \quad (14)$$

$$[R]_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_3 & sq_3 \\ 0 & -sq_3 & cq_3 \end{bmatrix}; \quad (15)$$

$$[R]_3^4 = \begin{bmatrix} cq_4 & 0 & -sq_4 \\ 0 & 1 & 0 \\ sq_4 & 0 & cq_4 \end{bmatrix}; \quad (16)$$

$$[R]_4^5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_5 & sq_5 \\ 0 & -sq_5 & cq_5 \end{bmatrix}; [R]_5^6 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (17)$$

3.2 The Angular Operational Velocities

Using the above presented initial data, the following relation (Detesan, 2007) will be used in order to determine the angular operational velocities of each link $(i), i = \overline{1,6}$, expressed into the frames (T_i):

$$\overline{\omega}_i^i = [R]_{i-1}^i \cdot \overline{\omega}_i^{i-1} + \begin{cases} \dot{q}_i \cdot \bar{k}_i^i, & \text{for rotation} \\ 0, & \text{for translation.} \end{cases} \quad (18)$$

The angular operational velocities are:

$$[\overline{\omega}]_1^1 = [R]_0^1 \cdot [\overline{\omega}]_0^0 + \dot{q}_1 \cdot [\bar{k}]_1^1 = \begin{bmatrix} cq_1 & sq_1 & 0 \\ -sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix}; \quad (19)$$

$$[\overline{\omega}]_2^2 = [R]_1^2 \cdot [\overline{\omega}]_1^1 + \dot{q}_2 \cdot [\bar{i}]_2^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_2 & sq_2 \\ 0 & -sq_2 & cq_2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix} + \begin{bmatrix} \dot{q}_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{q}_2 \\ \dot{q}_1 sq_2 \\ \dot{q}_1 cq_2 \end{bmatrix}; \quad (20)$$

$$[\overline{\omega}]_3^3=[R]_2^3\cdot[\overline{\omega}]_2^2+\dot{q}_3\cdot[\bar{i}]_3^3=\begin{bmatrix}1&0&0\\0&cq_3&sq_3\\0&-sq_3&cq_3\end{bmatrix}\cdot\begin{bmatrix}\dot{q}_2\\\dot{q}_1\,sq_2\\\dot{q}_1\,cq_2\end{bmatrix}+\begin{bmatrix}\dot{q}_3\\0\\0\end{bmatrix}=\begin{bmatrix}\dot{q}_2+\dot{q}_3\\\dot{q}_1\,s(q_2+q_3)\\\dot{q}_1\,c(q_2+q_3)\end{bmatrix};\qquad(21)$$

$$[\overline{\omega}]_4^4=[R]_3^4\cdot[\overline{\omega}]_3^3+\dot{q}_4\cdot[\bar{j}]_4^4=\begin{bmatrix}cq_4&0&-sq_4\\0&1&0\\sq_4&0&cq_4\end{bmatrix}\cdot\begin{bmatrix}\dot{q}_2+\dot{q}_3\\\dot{q}_1\,s(q_2+q_3)\\\dot{q}_1\,c(q_2+q_3)\end{bmatrix}+\begin{bmatrix}0\\\dot{q}_4\\0\end{bmatrix}=$$

$$=\begin{bmatrix}cq_4(\dot{q}_2+\dot{q}_3)-\dot{q}_1\,c(q_2+q_3)\cdot sq_4\\\dot{q}_4+\dot{q}_1\,s(q_2+q_3)\\sq_4(\dot{q}_2+\dot{q}_3)+\dot{q}_1\,c(q_2+q_3)\cdot cq_4\end{bmatrix};\qquad(22)$$

$$[\overline{\omega}]_5^5=[R]_4^5\cdot[\overline{\omega}]_4^4+\dot{q}_4\cdot[\bar{i}]_5^5=\begin{bmatrix}1&0&0\\0&cq_5&sq_5\\0&-sq_5&cq_5\end{bmatrix}\cdot\begin{bmatrix}cq_4(\dot{q}_2+\dot{q}_3)-\dot{q}_1\,c(q_2+q_3)\cdot sq_4\\\dot{q}_4+\dot{q}_1\,s(q_2+q_3)\\sq_4(\dot{q}_2+\dot{q}_3)+\dot{q}_1\,c(q_2+q_3)\cdot cq_4\end{bmatrix}+\begin{bmatrix}\dot{q}_5\\0\\0\end{bmatrix}=$$

$$=\begin{bmatrix}\dot{q}_5+cq_4(\dot{q}_2+\dot{q}_3)-\dot{q}_1\,c(q_2+q_3)\cdot sq_4\\sq_5(sq_4(\dot{q}_2+\dot{q}_3)+\dot{q}_1\,c(q_2+q_3)\cdot cq_4)+cq_5(\dot{q}_4+\dot{q}_1\,s(q_2+q_3))\\cq_5(sq_4(\dot{q}_2+\dot{q}_3)+\dot{q}_1\,c(q_2+q_3)\cdot cq_4)-sq_5(\dot{q}_4+\dot{q}_1\,s(q_2+q_3))\end{bmatrix};\qquad(23)$$

$$[\overline{\omega}]_6^6=[R]_5^6\cdot[\overline{\omega}]_5^5+\dot{q}_5\cdot[\bar{i}]_6^6=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}\dot{q}_5+cq_4(\dot{q}_2+\dot{q}_3)-\dot{q}_1\,c(q_2+q_3)\cdot sq_4\\sq_5(sq_4(\dot{q}_2+\dot{q}_3)+\dot{q}_1\,c(q_2+q_3)\cdot cq_4)+cq_5(\dot{q}_4+\dot{q}_1\,s(q_2+q_3))\\cq_5(sq_4(\dot{q}_2+\dot{q}_3)+\dot{q}_1\,c(q_2+q_3)\cdot cq_4)-sq_5(\dot{q}_4+\dot{q}_1\,s(q_2+q_3))\end{bmatrix}+$$

$$+\begin{bmatrix}\dot{q}_6\\0\\0\end{bmatrix}=\begin{bmatrix}\dot{q}_5+cq_4(\dot{q}_2+\dot{q}_3)-\dot{q}_1\,c(q_2+q_3)\cdot sq_4\\sq_5(sq_4(\dot{q}_2+\dot{q}_3)+\dot{q}_1\,c(q_2+q_3)\cdot cq_4)+cq_5(\dot{q}_4+\dot{q}_1\,s(q_2+q_3))\\cq_5(sq_4(\dot{q}_2+\dot{q}_3)+\dot{q}_1\,c(q_2+q_3)\cdot cq_4)-sq_5(\dot{q}_4+\dot{q}_1\,s(q_2+q_3))\end{bmatrix}.\qquad(24)$$

3.3 The Linear Operational Velocities

The linear operational velocities are determined using the following relation (Detesan, 2007):

$$\bar{v}_i^i=[R]_{i-1}^i\left\{\bar{v}_{i-1}^{i-1}+\overline{\omega}_{i-1}^{i-1}\times\bar{r}_i^{i-1}\right\}+\begin{cases}0&,\text{rotation}\\\dot{q}_i\cdot\bar{k}_i^i&,\text{translation}\end{cases}\qquad(25)$$

They correspond to the links $(i), i=\overline{1,6}$, they are expressed into the frames (T_i) and they have the following expressions:

$$[\bar{v}]_1^1=[R]_0^1\cdot\left\{\bar{v}_0^0+\overline{\omega}_0^0\times\bar{r}_1^0\right\}=\begin{bmatrix}cq_1&sq_1&0\\-sq_1&cq_1&0\\0&0&1\end{bmatrix}\cdot\left\{\begin{bmatrix}0\\0\\0\end{bmatrix}+\begin{bmatrix}0\\0\\0\end{bmatrix}\cdot\begin{bmatrix}0\\0\\0\end{bmatrix}\right\}=\begin{bmatrix}0\\0\\0\end{bmatrix};\qquad(26)$$

$$[\bar{v}]_2^2=[R]_1^2\cdot\left\{\bar{v}_1^1+\overline{\omega}_1^1\times\bar{r}_2^1\right\}=\begin{bmatrix}1&0&0\\0&cq_2&sq_2\\0&-sq_2&cq_2\end{bmatrix}\cdot\left\{\begin{bmatrix}0\\0\\0\end{bmatrix}+\begin{bmatrix}0&-\dot{q}_1&0\\\dot{q}_1&0&0\\0&0&0\end{bmatrix}\cdot\begin{bmatrix}0\\l_1\\l_2\end{bmatrix}\right\}=\begin{bmatrix}-l_1\dot{q}_1\\0\\0\end{bmatrix};\qquad(27)$$

$$[\bar{v}]_3^3=[R]_2^3\cdot\left\{\bar{v}_2^2+\overline{\omega}_2^2\times\bar{r}_3^2\right\}=\begin{bmatrix}1&0&0\\0&cq_3&sq_3\\0&-sq_3&cq_3\end{bmatrix}\cdot\left\{\begin{bmatrix}-l_1\dot{q}_1\\0\\0\end{bmatrix}+\begin{bmatrix}0&-\dot{q}_2cq_2&\dot{q}_1\,sq_2\\\dot{q}_2cq_2&0&-\dot{q}_2\\-\dot{q}_1\,sq_2&\dot{q}_2&0\end{bmatrix}\cdot\begin{bmatrix}0\\l_3\\0\end{bmatrix}\right\}=$$

$$=\begin{bmatrix}-\dot{q}_1(l_1+l_3cq_2)\\l_3\dot{q}_2sq_3\\l_3\dot{q}_2cq_3\end{bmatrix};\qquad(28)$$

$$[\bar{v}]_4^4 = [R]_3^4 \cdot \left\{ \bar{v}_3^3 + \bar{\omega}_3^3 \times \bar{r}_4^3 \right\} =$$

$$= \begin{bmatrix} cq_4 & 0 & -sq_4 \\ 0 & 1 & 0 \\ sq_4 & 0 & cq_4 \end{bmatrix} \cdot \left\{ \begin{bmatrix} -\dot{q}_1(l_1 + l_3 cq_2) \\ l_3 \dot{q}_2 sq_3 \\ l_3 \dot{q}_2 cq_3 \end{bmatrix} + \begin{bmatrix} 0 & -\dot{q}_1\, c(q_2+q_3) & \dot{q}_1\, s(q_2+q_3) \\ \dot{q}_1\, c(q_2+q_3) & 0 & -\dot{q}_2 - \dot{q}_3 \\ -\dot{q}_1\, s(q_2+q_3) & \dot{q}_2 + \dot{q}_3 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ l_4 \\ 0 \end{bmatrix} \right\} = \quad (29)$$

$$= \begin{bmatrix} -cq_4(\dot{q}_1(l_1 + l_3 cq_2) + l_4 \dot{q}_1 c(q_2+q_3)) - sq_4(l_4(\dot{q}_2 + \dot{q}_3) + l_3 \dot{q}_2 cq_3) \\ l_3 \dot{q}_2 sq_3 \\ cq_4(l_4(\dot{q}_2 + \dot{q}_3) + l_3 \dot{q}_2 cq_3) - sq_4(\dot{q}_1(l_1 + l_3 cq_2) + l_4 \dot{q}_1 c(q_2+q_3)) \end{bmatrix};$$

$$[\bar{v}]_5^5 = [R]_4^5 \cdot \left\{ \bar{v}_4^4 + \bar{\omega}_4^4 \times \bar{r}_5^4 \right\} =$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_5 & sq_5 \\ 0 & -sq_5 & cq_5 \end{bmatrix} \cdot \left\{ \begin{bmatrix} -cq_4(\dot{q}_1(l_1 + l_3 cq_2) + l_4 \dot{q}_1 c(q_2+q_3)) - sq_4(l_4(\dot{q}_2 + \dot{q}_3) + l_3 \dot{q}_2 cq_3) \\ l_3 \dot{q}_2 sq_3 \\ cq_4(l_4(\dot{q}_2 + \dot{q}_3) + l_3 \dot{q}_2 cq_3) - sq_4(\dot{q}_1(l_1 + l_3 cq_2) + l_4 \dot{q}_1 c(q_2+q_3)) \end{bmatrix} + \right.$$

$$\left. + \begin{bmatrix} cq_4(\dot{q}_2 + \dot{q}_3) - \dot{q}_1\, c(q_2+q_3) \cdot sq_4 \\ \dot{q}_4 + \dot{q}_1\, s(q_2+q_3) \\ sq_4(\dot{q}_2 + \dot{q}_3) + \dot{q}_1\, c(q_2+q_3) \cdot cq_4 \end{bmatrix} \cdot \begin{bmatrix} l_6 \\ l_5 \\ 0 \end{bmatrix} \right\} =$$

$$= \begin{bmatrix} \begin{array}{c} -cq_4(\dot{q}_1(l_1 + l_3 cq_2) + l_4 \dot{q}_1 c(q_2+q_3)) - sq_4(l_4(\dot{q}_2 + \dot{q}_3) + l_3 \dot{q}_2 cq_3) - \\ -l_5(sq_4(\dot{q}_2 + \dot{q}_3) + \dot{q}_1 c(q_2+q_3)cq_4 \\ \hdashline cq_5(l_6(sq_4(\dot{q}_2 + \dot{q}_3) + \dot{q}_1 c(q_2+q_3)cq_4) + l_3 \dot{q}_2 sq_3) - sq_5(l_6(\dot{q}_4 + \dot{q}_1 s(q_2+q_3) - \\ -cq_4(l_4(\dot{q}_2 + \dot{q}_3) + l_3 \dot{q}_2 cq_3) + sq_4(\dot{q}_1(l_1 + l_3 cq_2) + l_4 \dot{q}_1 c(q_2+q_3)) - \\ -l_5(cq_4(\dot{q}_2 + \dot{q}_3) - \dot{q}_1 c(q_2+q_3)sq_4) \\ \hdashline -cq_5(l_6(\dot{q}_4 + \dot{q}_1 s(q_2+q_3) - cq_4(l_4(\dot{q}_2 + \dot{q}_3) + l_3 \dot{q}_2 cq_3) + \\ + sq_4(\dot{q}_1(l_1 + l_3 cq_2) + l_4 \dot{q}_1 c(q_2+q_3)) - l_5(cq_4(\dot{q}_2 + \dot{q}_3) - \dot{q}_1 c(q_2+q_3)sq_4) - \\ -sq_5(l_6(sq_4(\dot{q}_2 + \dot{q}_3) + \dot{q}_1 c(q_2+q_3)cq_4) + l_3 \dot{q}_2 sq_3) \end{array} \end{bmatrix}. \quad (30)$$

Because the expressions of the linear operational velocity corresponding to the link 6 is very complex, it can be sent upon request to the email of the main author.

3.4 The Angular Operational Accelerations

The angular operational accelerations are determined using the following relation (Detesan, 2007):

$$\bar{\varepsilon}_i^i = [R]_{i-1}^i \cdot \bar{\varepsilon}_{i-1}^{i-1} + \\ + \begin{cases} [R]_{i-1}^i \cdot \bar{\omega}_{i-1}^{i-1} \times \dot{q}_i \cdot \bar{k}_i^i + \ddot{q}_i \cdot \bar{k}_i^i & \text{, rotation} \\ 0 & \text{, translation} \end{cases} \quad (31)$$

They are computed by the MATLAB script and they have the following expressions:

$$[\bar{\varepsilon}]_1^1 = [R]_0^1 \cdot [\bar{\varepsilon}]_0^0 + \left\{ [R]_0^1 \cdot \bar{\omega}_0^0 \times \dot{q}_1 \cdot \bar{k}_1^1 + \ddot{q}_2 \cdot \bar{k}_1^1 \right\} =$$

$$= \begin{bmatrix} cq_1 & sq_1 & 0 \\ -sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \left\{ \begin{bmatrix} cq_1 & sq_1 & 0 \\ -sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -\dot{q}_1 & 0 \\ \dot{q}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \dot{q}_1 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \ddot{q}_2 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 \end{bmatrix}; \quad (32)$$

$$\left[\bar{\varepsilon}\right]_2^2 = \left[R\right]_1^2 \cdot \left[\bar{\varepsilon}\right]_1^1 + \left\{\left[R\right]_1^2 \cdot \bar{\omega}_1^1 \times \dot{q}_2 \cdot \bar{i}_2^2 + \ddot{q}_2 \cdot \bar{i}_2^2\right\} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_2 & sq_2 \\ 0 & -sq_2 & cq_2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 \end{bmatrix} +$$
$$+ \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_2 & sq_2 \\ 0 & -sq_2 & cq_2 \end{bmatrix} \cdot \begin{bmatrix} 0 & -\dot{q}_1 & 0 \\ \dot{q}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \dot{q}_2 \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \ddot{q}_2 \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\} = \begin{bmatrix} \ddot{q}_2 \\ \ddot{q}_1\, sq_2 + \dot{q}_1 \dot{q}_2\, cq_2 \\ \ddot{q}_1\, cq_2 - \dot{q}_1 \dot{q}_2\, sq_2 \end{bmatrix}. \quad (33)$$

Because the expressions of the angular operational accelerations corresponding to the link 3-6 are very complex, they can be sent upon request to the email of the main author.

3.5 The Linear Operational Accelerations

The expression used to determine the linear operational accelerations is the following: (Detesan, 2007), valid only in the case of rotation joints:

$$\bar{a}_i^i = \left[R\right]_{i-1}^i \left\{\bar{a}_{i-1}^{i-1} + \bar{\varepsilon}_{i-1}^{i-1} \times \bar{r}_i^{i-1} + \bar{\omega}_{i-1}^{i-1} \times \left(\bar{\omega}_{i-1}^{i-1} \times \bar{r}_i^{i-1}\right)\right\}. \quad (34)$$

The linear operational accelerations are written in a matrix form as in the equations (35)-(36). Because the expressions of the linear operational accelerations corresponding to the link 3-6 are also very complex, they can be sent upon request to the email of the main author.

$$\left[\bar{a}\right]_1^1 = \left[R\right]_0^1 \cdot \left\{\bar{a}_0^0 + \bar{\varepsilon}_0^0 \times \bar{r}_1^0 + \bar{\omega}_0^0 \times \left(\bar{\omega}_0^0 \times \bar{r}_1^0\right)\right\} =$$
$$= \begin{bmatrix} cq_1 & sq_1 & 0 \\ -sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \left\{ \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\} = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}; \quad (35)$$

$$\left[\bar{a}\right]_2^2 = \left[R\right]_1^2 \cdot \left\{\bar{a}_1^1 + \bar{\varepsilon}_1^1 \times \bar{r}_2^1 + \bar{\omega}_1^1 \times \left(\bar{\omega}_1^1 \times \bar{r}_2^1\right)\right\} =$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_2 & sq_2 \\ 0 & -sq_2 & cq_2 \end{bmatrix} \cdot \left\{ \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \begin{bmatrix} 0 & -\ddot{q}_1 & 0 \\ \ddot{q}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ l_1 \\ l_2 \end{bmatrix} + \begin{bmatrix} 0 & -\dot{q}_1 & 0 \\ \dot{q}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & -\dot{q}_1 & 0 \\ \dot{q}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ l_2 \end{bmatrix} \right\} = \quad (36)$$
$$= \begin{bmatrix} -l_1 \ddot{q}_1 \\ g\, sq_2 - l_1\, \dot{q}_1^2\, cq_2 \\ l_1\, sq_2\, \dot{q}_1^2 + g\, cq_2 \end{bmatrix}.$$

3.6 Frame Transformations and the Unified Expressions of the Operational Kinematic Parameters

According to the following relations (Negrean, 2008):

$$\left[\dot{X}^n\right] = \left[\left[\bar{v}_n^n\right]^T \ \left[\bar{\omega}_n^n\right]^T\right]^T, \quad \left[\ddot{X}^n\right] = \left[\left[\bar{a}_n^n\right]^T \ \left[\bar{\varepsilon}_n^n\right]^T\right]^T. \quad (37)$$

the operational kinematic parameters can be expressed in a brief form as:

$$\left[\dot{X}^n\right] = \left[\left[\bar{v}_6^6\right]^T \ \left[\bar{\omega}_6^6\right]^T\right]^T, \quad \left[\ddot{X}^n\right] = \left[\left[\bar{a}_6^6\right]^T \ \left[\bar{\varepsilon}_6^6\right]^T\right]^T. \quad (38)$$

Using the transformation relations (Negrean, 2008):

$$\left[\dot{X}\right]^0 = \begin{bmatrix} \bar{v}_n^0 \\ -- \\ \bar{\omega}_n^0 \end{bmatrix} = \begin{bmatrix} \left[R\right]_n^0 & | & \left[0\right] \\ --- & | & --- \\ \left[0\right] & | & \left[R\right]_n^0 \end{bmatrix} \cdot \begin{bmatrix} \bar{v}_n^n \\ -- \\ \bar{\omega}_n^n \end{bmatrix} = \left[R\right]^0 \cdot \left[\dot{X}\right]^n; \quad (39)$$

$$\left[\ddot{X}\right]^0 = \begin{bmatrix} \bar{a}_n^0 \\ -- \\ \bar{\varepsilon}_n^0 \end{bmatrix} = \begin{bmatrix} \left[R\right]_n^0 & | & \left[0\right] \\ --- & | & --- \\ \left[0\right] & | & \left[R\right]_n^0 \end{bmatrix} \cdot \begin{bmatrix} \bar{a}_n^n \\ -- \\ \bar{\varepsilon}_n^n \end{bmatrix} = \left[R\right]^0 \cdot \left[\ddot{X}\right]^n, \quad (40)$$

the above determined kinematic parameters can be transformed from the frame (T_6) into the frame (T_0), from the robot's base. The matrix transformations are the following:

$$\left[\bar{v}\right]_6^0 = \left[R\right]_6^0 \cdot \left[\bar{v}\right]_6^6, \quad (41)$$

$$\left[\overline{\omega}\right]_6^0 = \left[R\right]_6^0 \cdot \left[\overline{\omega}\right]_6^6, \tag{42}$$

$$\left[\overline{a}\right]_6^0 = \left[R\right]_6^0 \cdot \left[\overline{a}\right]_6^6, \tag{43}$$

$$\left[\overline{\varepsilon}\right]_6^0 = \left[R\right]_6^0 \cdot \left[\overline{\varepsilon}\right]_6^6. \tag{44}$$

The unified notation for the operational velocities and accelerations, expressed into the frame (T_0) is the following:

$$\begin{aligned} \left[\dot{X}^0\right] &= \left[\left[\overline{v}_6^0\right]^T \ \left[\overline{\omega}_6^0\right]^T\right]^T, \\ \left[\ddot{X}^0\right] &= \left[\left[\overline{a}_6^0\right]^T \ \left[\overline{\varepsilon}_6^0\right]^T\right]^T. \end{aligned} \tag{45}$$

4 CONCLUSIONS

The equations (38) and (45) represent the equations of the direct kinematic model of the 5R articulated industrial robot, which can be used in welding technological processes. They express the operational velocities and accelerations, linear and angular, corresponding to the robot's attached tool, written with respect to the frames (T_6) and (T_0).

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