THE MODELLING OF INVOLUTE TEETH GENERATION, WITH THE RELATIVE GENERATING TRAJECTORIES METHOD

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ABSTRACT: The modelling of a curl of surfaces associated with a pair of rolling centrodes, when it is known the profile of the rack-gear's teeth profile, by direct measuring, as a coordinate matrix, has as goal the determining of the generating quality for an imposed kinematics of the relative motion of tool regarding the blank. In this way, it is possible to determine the generating geometrical error, as a base of the total error. The generation modelling allows highlighting the potential errors of the generating tool, in order to correct its profile, prior to using the tool in machining process. A method developed in CATIA is proposed, based on a new method, namely the method of "relative generating trajectories". The analytical foundation is presented, as so as an application for knows models of rack-gear type tools used on Maag teething machines.
KEY WORDS: profiling tools, generation by rolling, CATIA, trajectories method

1 INTRODUCTION

The method of relative generating trajectories, as complementary method for study of enwrapping surfaces, by rolling method, was presented (Teodor et al., 2015). A variant of this method was developed in the paper entitled "The Rack-Gear Tool Generation Modelling. Non-Analytical Method Developed in CATIA, Using the Relative Generating Trajectories Method", presented at IManEE Conference that was held at September 2016 in Chalkidiki, Greece. This paper represent the full version of the paper and includes a CATIA developed application, regarding the modelling of generation with rack-gear tool for an ordered curl of surfaces (profiles) associated with a pair of circular centrodes and also includes explanation regarding the graphical method and algorithm developed in graphical design environment.

The issue presented in this paper was approached in others forms by Baicu (Baicu, 2002), using the 3D modelling, as so as by Berbinschi and all. 2001, based on the "minimum distance" complementary theorem (Oancea, 2004). Also, the generation modelling is used in order to identify the optimal cutting scheme at generation with hob mill of the involutes teeth by Dimitriou and Antoniadis, 2008.

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At the same time, Teodor, 2010 developed an analytical form of the trajectories method, applied also for generation modelling. A variant of the "relative generating trajectories" method is presented in this paper, as complementary method for study of the enwrapping surfaces associated with a pair of rolling centrodes. Obviously, the complementary theorems are alternative ways for express the fundamental theorem of Willis (Litvin, 1984).

The graphical method proposed in this paper, applied for modelling of surfaces generated with the rack-gear tool, is based on the "relative generating trajectories" method. This method determines the trajectories of points belong to the tool's cutting edge in their relative motion regarding the blank.

The graphical expression is based on the specific algorithm, developed for a tool with profile known in discrete form, as an ordered cloud of points. This cloud of points is obtained by measuring of the generating tool. The graphical method is characterised by simplicity of application because of the CATIA system capabilities. The method is very rigorous and intuitive at the same time. The obtained results of the presented applications are compared with results obtained by an analytical method, in order to prove the graphical method's quality.

2 THE GENERATION MODELLING USING RACK-GEAR TOOL

The rolling centrodes, reference systems and profile of the rack-gear tool are presented in Figure 1.

They define the reference systems:

xy is the global reference system;

XY — the reference system joined with the blank and the C_1 centrode;

$\xi\eta$ — relative reference system joined with the C_2 centrode. The generating rack-gear's profile is joined with the C_2 centrode.

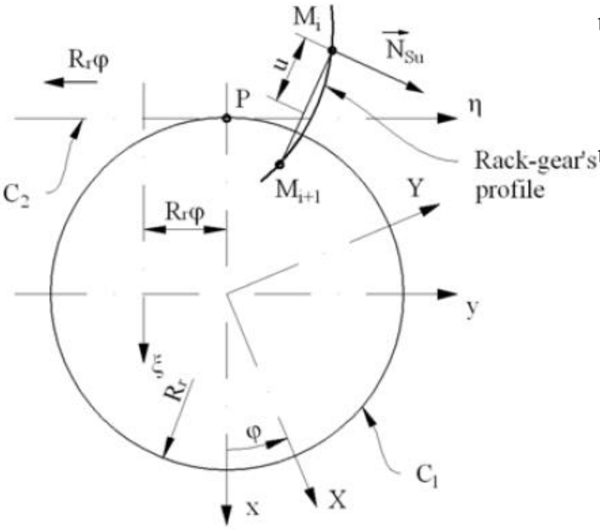


Figure 1. Centrodes and reference systems; the profile of rack-gear

The tool's profile is defined in the $\xi\eta$ reference system, in a discrete form,

$$S=\begin{pmatrix} \xi_1 & \eta_1 \\ \vdots & \vdots \\ \xi_i & \eta_i \\ \vdots & \vdots \\ \xi_n & \eta_n \end{pmatrix},\ i=1\ldots n. \tag{1}$$

For M_iM_{i+1} elementary segment which replace the rack-gear tool, with equations:

$$S_u\left|\begin{aligned} &\xi=\xi+u\cdot\cos\beta_i; \\ &\eta=\eta+u\cdot\sin\beta_i, \end{aligned}\right. \tag{2}$$

with,

$$\tan\beta_i=\frac{\Delta\eta}{\Delta\xi};\ \Delta\xi=\xi_{i+1}-\xi_i;\ \Delta\eta=\eta_{i+1}-\eta_i. \tag{3}$$

For i = 1 … n, with n significantly high, the profile of the rack-gear tool can be described very rigorous.

The $\vec{N}_{Su}$ normal versor to the elementary profile M_iM_{i+1},

$$\vec{N}_{Su}=\sin\beta_i\cdot\vec{i}+\cos\beta_i\cdot\vec{j} \tag{4}$$

is defined.

In this way, the parametrical equations of the normal to the elementary profile, in the point $M_i(\xi\eta)$, are given by

$$\vec{N}_{Su}\left|\begin{aligned} &\xi=\xi_i+u\cdot\cos\beta_i+\lambda\cdot\sin\beta_i; \\ &\eta=\eta_i+u\cdot\sin\beta_i+\lambda\cdot\cos\beta_i, \end{aligned}\right. \tag{5}$$

with λ as a scalar parameter. The direction of $\vec{N}_{Su}$ versor is given by (5).

In the relative motion between the rack-gear tool and the blank,

$$X=\omega_3(\varphi)\left[\xi+a\right];\ a=\begin{pmatrix} -R_r \\ -R_r\cdot\varphi \end{pmatrix}, \tag{6}$$

the normal (6) describes the family:

$$\left(\vec{N}_{Su}\right)_{\varphi}\left|\begin{aligned} &X=\left[\xi_i+u\cdot\cos\beta_i+\right. \\ &\left.+\lambda\cdot\sin\beta_i-R_r\right]\cdot\cos\varphi+ \\ &+\left[\eta_i+u\cdot\sin\beta_i+\right. \\ &\left.+\lambda\cdot\cos\beta_i-R_r\cdot\varphi\right]\cdot\sin\varphi; \\ &Y=-\left[\xi_i+u\cdot\cos\beta_i+\right. \\ &\left.+\lambda\cdot\sin\beta_i-R_r\right]\cdot\sin\varphi+ \\ &+\left[\eta_i+u\cdot\sin\beta_i+\right. \\ &\left.+\lambda\cdot\cos\beta_i-R_r\cdot\varphi\right]\cdot\cos\varphi. \end{aligned}\right. \tag{7}$$

In equations (7), u and φ are variable parameters.

For λ = 0, the equations (7) represents the trajectory family of a point from the rack-gear's profile, in the relative motion regarding the blank.

If the goal is to determine the S profile's enwrapping, it is necessary that the normals family $\left(\vec{N}_{Su}\right)_{\varphi}$ to pass through the gearing pole, P, for various rolling positions (Litvin, 1984). The different rolling positions are obtained for various values of the φ parameter.

If the gearing pole, P, is defined in the XY reference system,

$$P\left|\begin{aligned} &X_P=-R_r\cdot\cos\varphi; \\ &Y_P=R_r\cdot\sin\varphi, \end{aligned}\right. \tag{8}$$

then, from (7) and (8) results the equations assembly:

$$\begin{aligned} &\left[\xi_i+u\cdot\cos\beta_i-R_r\right]\cdot\cos\varphi+ \\ &+\left[\eta_i+u\cdot\sin\beta_i-R_r\cdot\varphi\right]\cdot\sin\varphi+ \\ &+\lambda\cdot\sin\left(\varphi+\beta_i\right)=-R_r\cdot\cos\varphi; \\ &-\left[\xi_i+u\cdot\cos\beta_i-R_r\right]\cdot\sin\varphi+ \\ &+\left[\eta_i+u\cdot\sin\beta_i-R_r\cdot\varphi\right]\cdot\cos\varphi+ \\ &+\lambda\cdot\cos\left(\varphi+\beta_i\right)=R_r\cdot\sin\varphi. \end{aligned} \tag{9}$$

The λ variable scalar is eliminated between the two conditions (9) and the enwrapping condition specifically for the relative generating trajectories is determined. This condition establish the blank's flank as enveloping for the trajectories family for points belong to the rack gear tool's flank, regarding the *XY* reference system of the blank,

$$\varphi = \frac{\left(\xi_i + u \cdot \cos\beta_i\right)\cdot\cos\beta_i}{-R_r \cdot \sin\beta_i} - \frac{\left(\eta_i + u \cdot \sin\beta_i\right)\cdot\sin\beta_i}{-R_r \cdot \sin\beta_i}. \tag{10}$$

The equations assembly:

$$\begin{aligned} X &= \left(\xi_i + u\cdot\cos\beta_i - R_r\right)\cdot\cos\varphi + \\ &+\left(\eta_i - u\cdot\sin\beta_i - R_r\cdot\varphi\right)\cdot\sin\varphi; \\ Y &= -\left(\xi_i + u\cdot\cos\beta_i - R_r\right)\cdot\sin\varphi + \\ &+\left(\eta_i - u\cdot\sin\beta_i - R_r\cdot\varphi\right)\cdot\cos\varphi, \end{aligned} \tag{11}$$

and the condition (10), determine the enwrapped profile — the analytical model of the blank's generated surface.

3 APPLICATION

The involute flank generated with a rack-gear tool is presented in Figure 2. In the same Figure are presented the pair of rolling centrodes and the reference systems are presented.

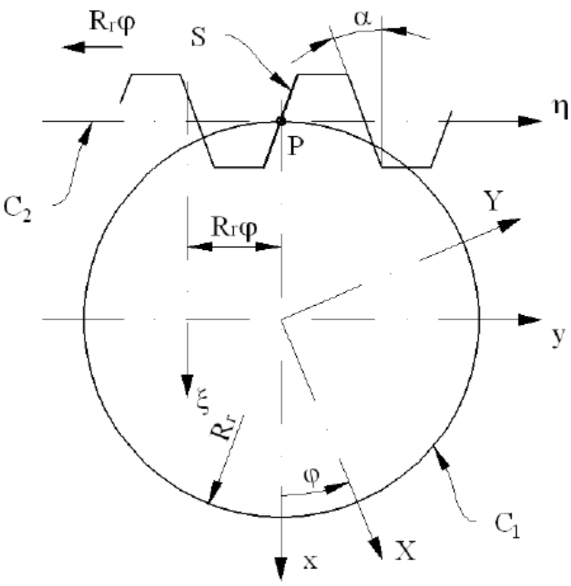


Figure 2. Rack-gear; reference systems

The *S* profile of the generating rack-gear was obtained in discrete form by measuring with a profile projector, see Figure 3. In Table 1, are presented the point's coordinates measured on the rack-gear's profile.

The equations assembly (11) and (10) determines the generating profile for the input data: $R_r = 50$ mm, rolling circle radius; $m = 5$ mm, modulus and $\alpha = 20°$.

Figure 3. The real rack-gear flank's measuring

Table 1. Model

Nr. crt.	X [mm]	Y [mm]
1	0.000	0.000
2	0.150	0.348
3	0.139	0.312
4	0.302	0.723
5	0.690	1.722
6	1.040	2.724
7	1.357	3.721
8	1.708	4.725
9	2.032	5.721
10	2.380	6.724
11	2.742	7.720
12	3.086	8.725
13	3.430	9.721
14	3.783	10.726
15	4.148	11.726
16	4.511	12.722
17	4.660	13.250

In Figure 4 depicts the generated profile of an involute tooth, the analytical modelled profile. A graphical solution was developed in CATIA design environment. According to this method, in DMU Kinematics module, the generating process was simulating by rolling for the tooth's flank, using the rack-gear tool.

The rack-gear-measured flank was modelled using a spline curve that has as control points the points with coordinates measured using the profile projector.

In the same file a normal line to the tool's profile (to the spline curve) was drawn which was constrained to pass through one of the control points.

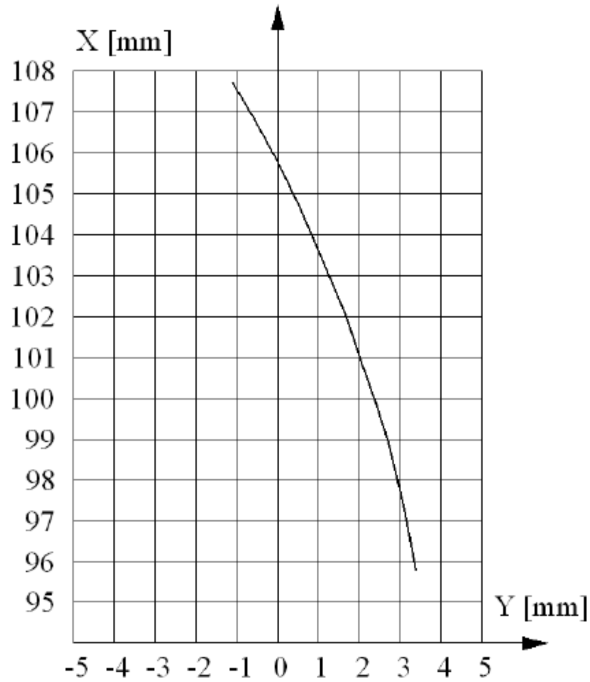


Figure 4. The modelled profile

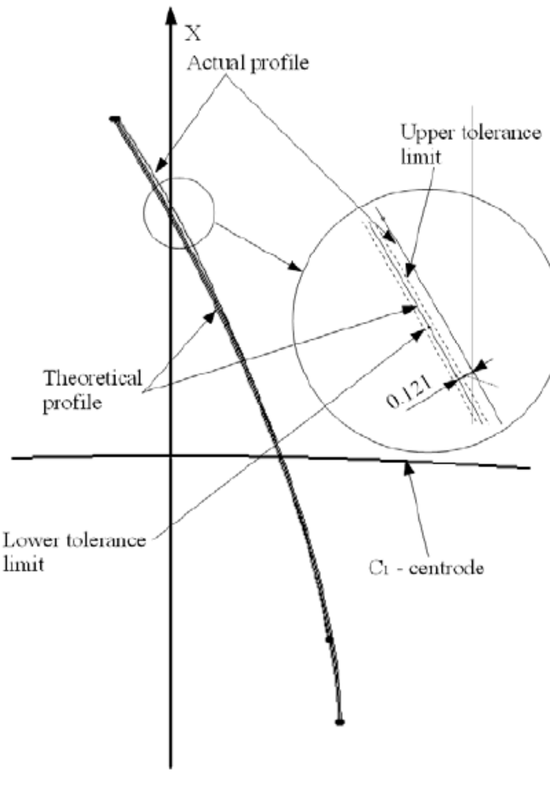


Figure 5. The generated profile

By simulating the mechanism function, keeping as fixed element the rack-gear, the position is found when the previously drawn normal line pass through the gearing pole. The intersection point between this normal and the modelled profile will be a point from the generated involute profile.

This last step is repeated for each of the measured points, obtaining points onto the generated profile.

Because the deviation of the rack-gear tool's profile exceeds the allowed limits, we can expect that the deviations of the tooth's profile exceed the allowed limits too. In Figure 5 are presented the limits obtained for the given input data.

4 GRAPHICAL METHOD DEVELOPED IN CATIA

In order to determine the generating errors induced by deviations from the tool's theoretical profile, the method of "inverse generation" is applied. According to this method, it starts from a known profile of tool (determined by measuring) and determines the profile of piece generated by this tool.

For this, the roles of piece and tool are changed, analysing the specific kinematics for generation with rotary cutter.

For graphical purpuses, in CATIA, a product file which will contain an assembly composed is created three elements and named *RackGear*.

The first element is itself a subassembly that contains two components on type "part".

The first part is the piece's centrode that, in this case, is a circle with radius R_r, and *XYZ* reference system's axis and origin.

The second part will contain the points determined onto the piece's profile and consequently, the profile of piece. The piece's profile will be a spline that has as control points the above-mentioned points.

The second element is also a subassembly composed from a part with is the tool's centrode, in this case a line, as so as, the $\xi\eta\zeta$ reference system's axis and origin.

The second part of this element is represented by the tool's profile, discretely knows, by measuring. The profile of tools is drawn as a spline that admits as control points those measured on profile projector.

A current point is considered as point on curve corresponding to each of the measured control points. On this point a line is drawn perpendicular to the curve that represent the tool's profile.

The third element, named *GlobalRS*, contains the elements of the global reference system. Here is defined the gearing pole. The role of this element is to assure the relative position between piece and tool.

In this assembly, using the *DMU Kinematics* module of the CATIA environment, a rack joint mechanism is defined (Figure 6)

The rack joint is composed by a prismatic joint and a revolute joint.

The prismatic joint is defined between the η axis of the *ToolSubassembly* product and a line parallel to *y* axis of the *GlobalRS* product. In addition, for this joint is define the coincidence between *xy* planes of the *ToolSubassembly* and *GlobalRS* products.

The revolution joint is defined between the *Z* axis of the *PieceSubassembly* product and the *z* axis of the *GlobalRS* product. The coincidence between the *xy* plane of the *PieceSubassembly* product and the similar plane of the *GlobalRS* product is imposed.

This mechanism has as fixed element the *ToolSubassembly* product and as drive element, the rotation angle of piece's reference system around the *Z* axis. Still here is determined and kept the distance measured from the gearing pole to the normal that passes through the current point, see Figure 7.a.

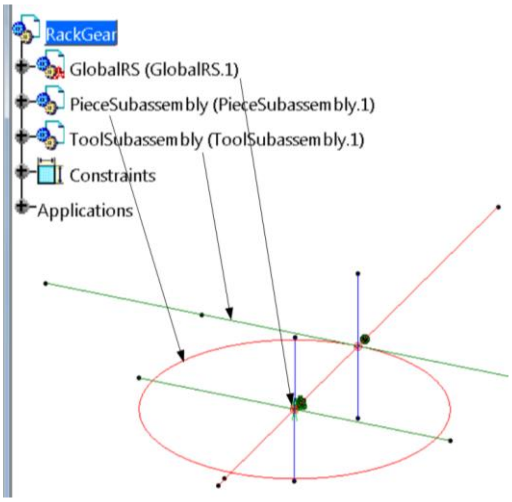
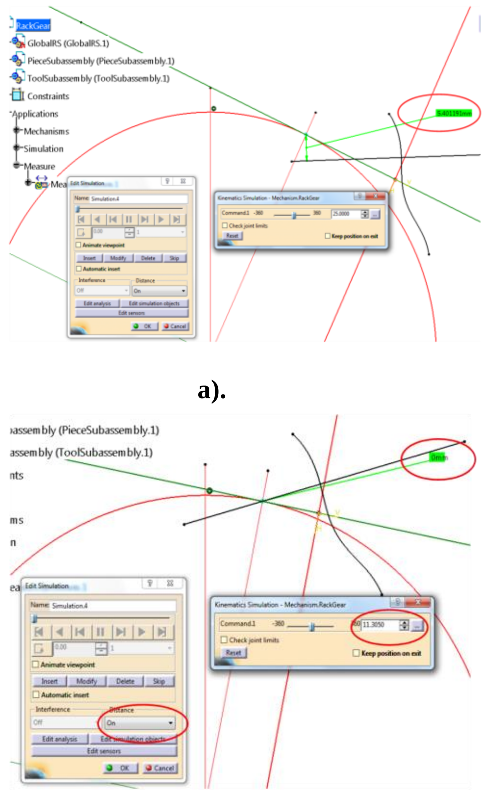


Fig. 6. RackGear assembly

In order to indentify the contact points between piece and tool we will apply the following algorithm:

1). The mechanism simulation is made, using the *SIMULATION* command and the position where the normal to the profile pass through the gearing pole is identified. The identification is made by monitoring the distance between the gearing pole and the line with the direction perpendicular to the profile.

Modifying the rotation angle and the step of this angle, after a few attempts, the coincidence between the normal and the gearing pole is obtained, see Figure 7.b. When the distance from the gearing pole to the normal become lower than a certain value (established by user, usually 10^{-3}—10^{-4} mm) the simulation is stopped and the mechanism position is keeped.



a).

b).

Figure 7. Distance between gearing pole and normal to the profile: a). initial position; b). final position

2). With the mechanism stopped in this position, the piece's profile is activated and the intersection point is determined between the normal to the profile and the tool's profile (Figure 8)

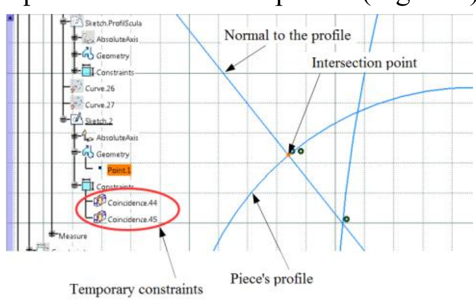


Figure 8. Intersection point between normal and profile; temporary constraints

For a correct positioning, two temporary constraints are applied, so the point belongs at the same time to the normal and to the profile (point-normal constraint: *constraint definition* ➔ *coincidence*, point-profile constraint: *constraint definition* ➔ *coincidence*).

Consequently, in order to keep the point's position when the mechanism position is changed, these two constraints are deleted and the point is fixed using the *isolate* command.

3). The tool's profile is activated and the normal is defined as passing through the next control point of the tool's profile (point determined by measuring with the profile projector). In this way, the normal's position is changed according to a new position onto the tool's profile.

The first step is resumed for the new position of normal and the algorithm continues with steps 2 and 3, until the number of points is determined (enough to drawn the piece's profile).

The piece's profile is drawn as a spline curve that admits as control points all the points previously determined.

5 CONCLUSIONS

The relative generating trajectories method is a complementary method for study of the enwrapping surfaces. The presented applications refers to conjugated profiles associated with a pair of rolling centrodes — the particular case of the generation with rack-gear regarding the generation starting from the discrete form of the tool's profile.

The graphical solution developed in CATIA use the capabilities of this design environment.

The analytical results and those obtained with the graphical algorithm are identical from the technical point of view. The graphical method is simple, intuitive and easy to apply and, in addition aloe to avoid rough errors which may be difficult to detect using only analytical methods. The generating trajectories may highlight the problem linked with the profile's interference at machining.

6 ACKNOWLEDGMENTS

This work was supported by a grant of the Romanian National Authority for Scientific Research and Innovation, CNCS-UEFISCDI, project number PN-II-RU-TE-2014-4-0031.

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