

APPLICATION AND RESEARCH OF DIFFERENTIAL GEOMETRY IN CONTROL SYSTEM

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ABSTRACT: Differential geometry is becoming increasingly used in industrial production. It has been transformed to industrial methods concerning with its mathematical theories, and been applied in many industrial control systems. We present a comprehensive study from various domains. At first, as to the domain of chaotic system, we take Lorenz and Rossler as examples, use differential geometric method and design a controller to control these two systems. Secondly, as to the domain of decoupling control, we study tilting control on the steering vehicle body, and construct motion model of steering vehicle through symbolic operation based on differential geometry. What's more, as to the domain of unmanned aerial vehicle (short as UAV) technology, we integrate geometrical mechanics with differential geometry and put forward a geometrical and mechanical method to model UAV. The validity and feasibility of these methods are verified through simulation experiments.

KEY WORDS: differential geometry; chaotic system; vehicle steering; Unmanned Aerial Vehicle (UAV) technology.

1 INTRODUCTION

When chaotic phenomenon arises in a system, it is difficult to analyse the system. People have proposed many methods to control chaotic phenomenon, but along with the development of research, some mathematical methods have been recognized and promoted, especially differential geometric method, which has good effect in chaotic control. Via deducing the differential geometry, many principles based on spatial geometry analysis have been proposed, such as the famous Leibniz principle and Lie derivative principle. Leibniz principle reveals the relationships between indefinite integrals with parameters. Lie derivative principle can be seen as a process of derivation, which is similar to the popular curve derivation. The tangent vector value at the target node can be seen as the slope coefficient of target node to the tangent. In general, if the slope coefficient is available, it will be very easy to work out the tangent equation.

Passive suspension system, boasting of numerous advantages such as simple structure, economic price and reliable quality, has been increasingly used in the suspension system of vehicles. However, the spring rigidity and damper coefficient of these suspensions are designed based on specific requirement, so they cannot be adjusted or change with signals. They can't meet various work conditions.

To increase the comfort of vehicles, especially driving steadiness, people have invested a large sum of money and manpower to research and develop active suspension systems.

Applying differential geometry and classical mechanics geometry to UAV technology is of great value of research and prospect of application. Geometric mechanics redefines classical mechanics from the perspective of differential geometry. In substance, the motion trajectory of UAV is similar to a three-dimensional curve. The moving of UAV on the trajectory is like the appearance change of the frame when moved on the curve. The turning and climbing of the UAV can be transformed to the curvature and torsion of the curve. Based on these understandings, the operation process of UAV can be simulated through the three-dimensional differential curve. UAV is always seen as a mechanics system under control, which can be described through differential equation, and the control functions involved can be expressed with parameters.

2 LITERATURE REVIEW

At present, most of the researches are concentrated on the object of differential geometric research on nonlinear feature, condition stimulation of moving objects, and describing systems with differential equations concerning with other subjects, etc. In literature [1], the theories of differential geometry and control system are

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integrated and nonlinear control theory is constructed. In literature [2], a nonlinear space vehicle is developed from the respect of wide-angle polynomial feedback control, which consists of five devices: a non-symmetry space vehicle and four wheels. In literature [3], differential geometry is integrated with PID technology to put forward a nonlinear excitation controller with temporary stability. The researcher connected the output of PID controller with that of the linear multi-variable excitation controller. In literature [4], the author added another two modules on the basis of the previous three-dimensional Lorenz module, which has become a five-dimensional Lorenz module. Thus to decide and judge their roles on different conditions. In literature [5], based on the differential geometric feedback first-order theory, a nonlinear decoupling control method of three-phase voltage source PWM rectifier is proposed.

In literature [6], the author emphasizes to apply the theories of differential geometry and mechanics to chaotic system. Local metric properties of curvature and torsion offer the analytical expression of slow manifold equation in the slow and fast autonomous discrete dynamical systems. In literature [7], researcher realizes the exact linearization of nonlinear vehicle suspension system module with differential geometric method. In literature [8], concerning with decoupling subsystem of differential meometric method, a attenuating control rule is designed to reduce the vibration of decoupling subsystem. In literature [9], a polynomial hysteresis damper module is applied to realize the control on magneto rheological damper . In literature [10], the moving of vehicle suspension system is decoupled again. To transfer the raising and pitching in vibrating as well as turning of the spring to linear subsystem. In literature [11], based on differential geometry, the nonlinear system connected with the hydraulic gas suspension can be transformed to linear system. In literature [12], a magneto rheological damper based on DC voltage control is analyzed and applied to the vehicle suspension system. In literature [13], a hilf-vehicle nonlinear model with active suspension is proposed, applying differential geometric method for decoupling. In literature [14], a abstract guidance law is proposed to serve the assembling and collision-avoidance of UAV. In literature [15], targeted at a moving object, the guidance law about the bout, confrontation and tracking of UAV is put forward. In literature [16], based on differential geometry, the author proposes a calculation method aiming at the detection and extinction of the

invaders. In literature [17], referring to the differential and geometric control method, a landing module of steel rope space vehicle is designed, whose degree of freedom is 8, but 4 without driving. In literature [18], based on the cubic visual perception on the obstacles, an effective calculation method to avoid conflicts is proposed.

3 METHODS

3.1 Control on the stability of Lorenz system and Rossler system

To transform nonlinear system to linear system through differential geometric method. The system is controllable when it's linear. Define the equation of Lorenz chaotic system as:

$$\begin{cases} Lq = F(Lx, Ly, Lz) \\ Lx = u(y - x) \\ Ly = ex - xz - y \\ Lz = xy - fz \end{cases} \quad (3.1)$$

In this equation, x refers to the intensity of convector, y refers to the temperature difference of convector, and z is a parameter to set the non-linearity. We've set different valus here to stimulate different systematic conditions. When $u = 10, e = 27, f = 81/31$, if we need the stability of Lorenz system, modify the equation to:

$$x = W(x) + R(x)q, y = \beta(x), \quad (3.2)$$

$$W = \begin{cases} x1 = u(x2 - x1) \\ x2 = fx1 - x1x3 - x2 \\ x3 = x1x2 - ex3 \end{cases}, R = \begin{bmatrix} t1 \\ t2 \\ t3 \end{bmatrix} \quad (3.3)$$

According to the principle of Frobenius, when $x = 0$ in $[s, ad_f s, ad^2_f s]$, the rank is 3. At the same time, when $x = 0$ in $[s, ad_f s, [s, ad_f s]]$, the rank is 2. Thus the value ranges of parameter t is worked out. The vector group of parameter is set as $t1 = 0, t2 = t3 = 1$, according to:

$$(a\beta_j(x) / ax).W_i(x) <= d\beta_j, W_i >= 0 \quad (3.4)$$

We can draw the equation about β

$$\beta = x1^2 - fx1 + x1 + ux2 - ux3 \quad (3.5)$$

Through differential and geometric transformation, the equation of Lorenz system is redefined as:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} \beta_1(x) \\ L_f \beta(x) \\ L_{f^2} \beta(x) \end{pmatrix} \quad (3.6)$$

And then, we can work out that:

$$\begin{aligned} z_1 &= x_1^2 - fx_1 + x_1 + ux_2 - ux_3; \\ z_2 &= (ue - u - uf)x_1 - ufx_2 + ufx_3 - \\ &\quad - 2ux_1^2 - ux_1x_3 + ux_1x_3; \\ z_3 &= u^2x_2^2 + (u^2e - u^2 - u^2f + uf)x_2 + \\ &\quad + (uf - u - 5u^2)x_1x_2 - u^2x_2x_3 - \\ &\quad - (u^2e - u^2 - u^2f + u^2ef)x_1 + (ue - \\ &\quad - 4u^2)x_1^2; \end{aligned} \quad (3.7)$$

Based on differential and geometric rule, the equation of stable Lorenz system is:

$$U = (1/L_R L_{W^2} \beta(x))(-L_{W^3} \beta(x) + T_3) \quad (3.8)$$

Move on the research on the control domain of differential geometric under nonlinear condition. We can see that in the practical production, nonlinear devices are used almost everywhere, such as the research on rules of weather changes, the historical process of biological evolution, the construction of protein and many other fields. All of these need to be described with nonlinear theories. Chaotic system is a main aspect of nonlinear system. Along with the research, it is also classified to time chaos and spatial chaos, on which many researchers have proposed various control and synchronous methods.

From the mechanics of Rossler chaotic system, define the equation as:

$$\begin{cases} \dot{R}q = F(Rx, Ry, Rz) \\ \dot{R}x = -x_2 - x_3 \\ \dot{R}y = x_1 + ux_2 \\ \dot{R}z = f + x_1x_3 - ex_3 \end{cases} \quad (3.9)$$

In these equations, u,e and f are there parameters of the system, when setting $u = 0.2, f = 0.2, c = 5.7$, modify the equation of Rossler chaotic system to:

$$W = \begin{cases} x_1 = -x_2 - x_3 \\ x_2 = x_1 + ux_2, \\ x_3 = f + x_1x_3 - ex_3 \end{cases} \quad R = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} \quad (3.10)$$

According to the Frobenius rule, when $x = 0$ in $[s, ad_f s, ad^2_f s]$, the rank is 3, and meanwhile, when $x = 0$ in $[s, ad_f s, [s, ad_f s]]$, the rank is 2. Because:

$$(a\beta_j(x)/ax).W_i(x) = d\beta_j, \quad W_i \geq 0,$$

we can draw the vector group of R is $(t_1x_3, 0, 0)$.

Based on differential geometry, transfer the equation of Rossler chaotic system to:

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} \beta(x) + 0(x) \\ L_f \beta(x) + 0(x) \\ L_{f^2} \beta(x) + 0(x) \end{bmatrix} \quad (3.11)$$

Seeing from the transformed equation, we can see that its result is almost same to that of the Lorenz system. The only difference is infinitesimal value $0(x)$.

Based on differential and geometric rule, the equation of stable Rossler system is:

$$U = (1/L_R L_{W^2} \beta(x))(-L_{W^3} \beta(x) + T_3) \quad (3.12)$$

3.2 Active suspension module of steering vehicle

The suspension system and four wheels are coupling with each other when the vehicle is steering, the reason for which is that tilting has broken the balance of torque. According to the statistical analysis from NHSA, the larger the angle of wheels, the greater the tilting radian. The annual percentages of hurt and death resulting from rolling over are 30% and 22% respectively.

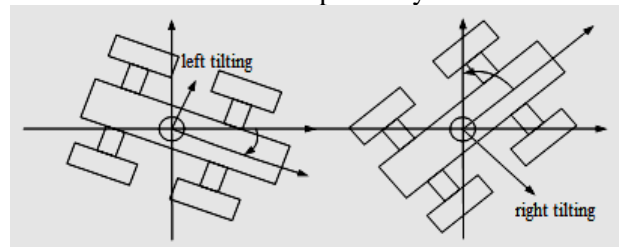


Fig. 1 Vehicle steering and tilting

Construct mathematical module, use symbolic parameter to express, and conduct decoupling on the moving of vehicles with differential and

geometric theories. Through decoupling, weaken the items in the equation which are connected with corner, to relieve the influence of vibration.

Considering the tilting of vehicle body, the equation of steering refers to:

$$\begin{cases} Mu(\theta + V_z) = Q_{yf} + Q_{yr} + M_s h \alpha \\ I_z V_z = x Q_{yf} - a Q_{yr} \end{cases} \quad (3.13)$$

In the equation, u is the driving speed, θ is the tilting radian of vehicle body, M is the mass of the vehicle, I_z is the inertial vector when the yaw is rotating, V_z is the angular velocity when steering, x is the distance between center of vehicle mass and fore shaft, a is the distance between center of vehicle mass and back shaft, h is the distance between center of vehicle mass and tilting center, Q_{yf} is the total tilting torque of the fore wheel, and Q_{yr} is the tilting torque of the back wheel.

The floating motion equation of the mass above the spring is:

$$\begin{cases} I_y \beta = (Q_{s4} + Q_{s4}) a \cos \beta - \\ -(Q_{s1} + Q_{s2}) x \cos \beta \end{cases} \quad (3.14)$$

In this equation, I_y is the rotating vector of the floating of mass above spring, Q_{s1} , Q_{s2} , Q_{s3} and

$$Q_{s1} = k_{t1}(Z_{c1} - Z_{s1}) + C_{s1}(Z_{c1} - Z_{s1}) - (K_{af} / 2d)(TR - [(Z_{c2} - Z_{c1}) / 2d]) + u1 \quad (3.18)$$

$$Q_{s2} = k_{t2}(Z_{c2} - Z_{s2}) + C_{s2}(Z_{c2} - Z_{s2}) + (K_{af} / 2d)(TR - [(Z_{c2} - Z_{c1}) / 2d]) + u2 \quad (3.19)$$

$$Q_{s3} = k_{t3}(Z_{c3} - Z_{s3}) + C_{s3}(Z_{c3} - Z_{s3}) + (K_{ar} / 2d)(TR - [(Z_{c3} - Z_{c4}) / 2d]) + u3 \quad (3.20)$$

$$Q_{s4} = k_{t4}(Z_{c4} - Z_{s4}) + C_{s4}(Z_{c4} - Z_{s4}) - (K_{ar} / 2d)(TR - [(Z_{c3} - Z_{c4}) / 2d]) + u4 \quad (3.21)$$

When the floating angle β and tilting radian TR are all within a certain range, we can draw the approximate value.

$$\begin{aligned} Z_{s1} &= Z_s + x\beta - dTR \\ Z_{s2} &= Z_s + x\beta + dTR \\ Z_{s3} &= Z_s - a\beta + dTR \\ Z_{s4} &= Z_s - a\beta - dTR \end{aligned} \quad (3.22)$$

These are approximate values, among which, Z_s is the displacement of center of mass above the spring, Z_{si} is the displacement of mass above the spring, is the degrees of elasticity of spring above the suspension, C_{si} is the damper coefficient, ui is

Q_{s4} are all active suspension torque, and β is the floating angle.

The equation of the tilting of mass above the spring is:

$$I_x TR = M_s u (\theta + V_z) h + M_s g h \sin TR + (Q_{s1} + Q_{s4} - Q_{s2} - Q_{s3}) d \cos TR \quad (3.15)$$

In this equation, I_x is the rotating vector of mass tilting of the mass above spring, TR is the tilting radian, d is a half of the distance between the left and right wheels.

The longitudinal equations of motion of the mass above spring is:

$$M_s Z_s = Q_{s3} + Q_{s4} + Q_{s1} + Q_{s2} \quad (3.16)$$

The longitudinal equations of motion of the mass below spring is:

$$M_{ci} Z_{ci} = k_{ti}(N_i - Z_{ci}) - Q_{si}, i = 1, 2, 3, 4 \quad (3.17)$$

In this equation, Z_{ci} is the displacement of mass below the spring, M_{ci} is the mass below the spring, N_i refers to the tire pressure from the ground, and k_{ti} is the tire rigidity.

In this way, we can draw every active suspension torque:

the torque under a certain speed, K_{af} and K_{ar} refer to the rigidity of lateral stability leverage in front of and behind the suspension.

When thinking about the torque module of the tire, postulate the cornering angle of the tire is relatively small, we can say that the performance of the tire is linear.

Considering with the tilting factors of vehicle body, the tilting torque of the tire can be expressed as:

$$Q_{yf} = j_1 \alpha_1 = a c_1 (p t - \theta - x V_z / u + a l_f TR) \quad (3.23)$$

$$Q_{yr} = j_2 \alpha_2 = a c_2 (a V_z / u - \theta + a l_r TR) \quad (3.24)$$

In the equation, j_1 and j_2 are the tilting rigidity of the fore wheel and the back wheel, α_1 and α_2 are the tilting radian of the fore wheel and the back wheel, al_f and al_r refer to the steering factor of the fore and back wheels, and pt is the the turning degree of the fore wheel.

3.3 UAV geometric mechanics modeling of Newtonian forms

The UAV model is constructed on the basis of rigid motion group, which refers to the subgroup of special European group SE(3). The corresponding group element Fab consists of the translational component and rotational component Rab which is the subject coordinate system b to the spatial coordinate system a . SE(3) involves a sub-aggregate SO(3), which is called special orthogonal group.

From the perspective of kinetics and dynamics, we apply differential geometric method to model UAV.

Set the navigation equation of the space vehicle as:

$$(x_o, y_o, z_o) = Wab(v)(uc, vc, wc) \quad (3.25)$$

In this equation, x_o, y_o, z_o refer to the vector component of north, east and vertical direction of the space vehicle in the geographic coordinate system respectively. The space velocity of the vehicle $uc, vc,$ and wc under the plane system are set as the X-axis direction component, Y-axis direction component, and Z-axis direction component respectively. Here, $Wab(v)$ refers to a function, referring to XYZ euler rotation from body frame to local geography.

$$Wab(v) = \begin{bmatrix} A\alpha A\theta & -A\beta b\theta + b\beta b\alpha A\theta & b\beta b\theta + A\beta b\alpha A\theta \\ A\alpha b\theta & A\beta A\theta + b\beta A\alpha b\theta & -b\beta A\theta + A\beta b\alpha b\theta \\ -b\alpha & b\beta A\alpha & A\beta A\alpha \end{bmatrix} \quad (3.26)$$

In the above equations, ‘ A ’ refers to solving by cos , ‘ b ’ refers to solving by sin . Set the matrix $[x_o, y_o, z_o]^T$ and $[uc, vc, wc]^T$ as variable $EW_{ab}^a(v)$ and $ES_{ab}^b(v)$, then the equation (3.26) can be modified to:

$$EW_{ab}^a(v) = Wab(v).ES_{ab}^b(v) \quad (3.27)$$

Based on (3.26) and (3.27), we can deduce the kinetics equation of the space vehicle:

$$\begin{bmatrix} \beta \\ \alpha \\ \theta \end{bmatrix} = \begin{bmatrix} pe + \tan\alpha(qe\sin\beta + re\cos\beta) \\ qe\cos\beta - re\sin\beta \\ (qe\sin\beta + re\cos\beta) / \cos\alpha \end{bmatrix} \quad (3.28)$$

In the equation, $\beta, \alpha,$ and θ are intersection angle, floating angle, and lane departure angle respectively. $pe, qe,$ and re refer to rotation around X-axis, the floating speed of the axis and the lane departure speed of the Z-axis.

Use $V_{ab}^b(t)$ to express $[pe, qe, re]^T$, introduce arithmetic operators $\hat{\cdot}$. Random parameter $rw \in R^3$ meets the vector production relationship $V_{ab}^b(t).rw = V_{ab}^b(t) \times rw$. We can express $V_{ab}^b(t)$ as the matrix involving $pe, qe,$ and re , and then draw out that:

$$Wab(v) = Wab(v).V_{ab}^b(t) \quad (3.29)$$

So, the kinetics model

$Fab(t) = Fab(t).VL_{ab}^b(t)$ constructed based on SE(3), express it with matrix:

$$\begin{bmatrix} \dot{Wab}(v) \\ \dot{EW}_{ab}^a(v) \\ \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} \dot{Wab}(v) \\ \dot{EW}_{ab}^a(v) \\ \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \dot{V}_{ab}^b(t) \\ \dot{ES}_{ab}^b(v) \\ \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (3.30)$$

From the perspective of mechanics, the equation of the space vehicle can be expressed as:

$$\begin{bmatrix} u^* \\ v^* \\ w^* \end{bmatrix} = \begin{bmatrix} rev - qew \\ pew - reu \\ qeu - pew \end{bmatrix} + \begin{bmatrix} -\sin\alpha \\ \sin\beta.\cos\alpha \\ \cos\beta.\cos\alpha \end{bmatrix} g + \begin{bmatrix} [(az - s) / M].Cxt \\ [(az - s) / M].Cyt \\ [(az - s) / M].Czt \end{bmatrix} + \begin{bmatrix} Te / M \\ 0 \\ 0 \end{bmatrix}$$

(3.31)

According to (3.31), u^*, v^*, w^* is the component of X-axis, Y-axis and Z-axis of the space vehicle accelerated velocity to the plane

system respectively. M is the space vehicle mass, s is the wing area, az refers to the pressure resulting from air flow, and Te is the boost power of

the motor, and parameter Cxt , Cyt , and Czt are the bearing force coefficient of X Y and Z-axis respectively. We can simplify this equation, use $FB_A^b(t)$ and $FB_T^b(t)$ to express

$(az.s.Cxt, az.s.Cyt, az.s.Czt)^T$, and $[T, 0, 0]^T$ respectively. The former is the aerodynamic force, and the later is the boost power of the plane motor. We can modify (3.31) to:

$$M \cdot ES_{ab}^b(v) = -M \cdot V_{ab}^b(t) \times ES_{ab}^b(v) + W^T ab(v) e3(Mg) + FB_A^b(t) + FB_T^b(t) \quad (3.32)$$

Deduce the torque equation of the space vehicle, use variable JRx , JRy , and JRz to express the rotating inertial vector of X axis, Y axis, and Z axis. We need to draw the inertial vector $JRxz$ under XZ three-dimensional plane. Use $CLlt$, $CLmt$, and $CLnt$ to express the corresponding rotating torque coefficient, floating torque coefficient, and lane deprature torque coefficient respectively. Use bt to express the wing length of the plane.

$$\begin{bmatrix} JRx & 0 & -JRxz \\ 0 & JRy & 0 \\ -JRxz & 0 & JRz \end{bmatrix}$$

We use variable J to express the above parameter matrix and to be the inertial vector matrix of the space vehicle.

Therefore, the dynamics model of UAV can be expressed as:

Turn the parameter to matrix:

$$\begin{bmatrix} V_{ab}^b(t) \\ ES_{ab}^b(v) \end{bmatrix} = \begin{bmatrix} J & 0 \\ 0 & MI3 \end{bmatrix} \left(\begin{bmatrix} JV_{ab}^b(t) & 0 \\ 0 & -MV_{ab}^b(t) \end{bmatrix} \begin{bmatrix} V_{ab}^b(t) \\ ES_{ab}^b(v) \end{bmatrix} + \begin{bmatrix} TBV(T) \\ MgW^T ab(v) e3 + FB_A^b(t) + FB_T^b(t) \end{bmatrix} \right) \quad (3.33)$$

In this matrix, we express

$$[az.s.b(CLlt + CLnt), az.s.b.CLmt, az.s.b(CLlt + CLnt)]^T \text{ as } TBV(T).$$

4 RESULT AND DISCUSION

We conduct experimental analysis through MatLab from two aspects:

To testify the Lorenz chaotic system is becoming more and more stable under the effectiveness of the controller, we conduct numerical simulation to testify the feasibility of this method.

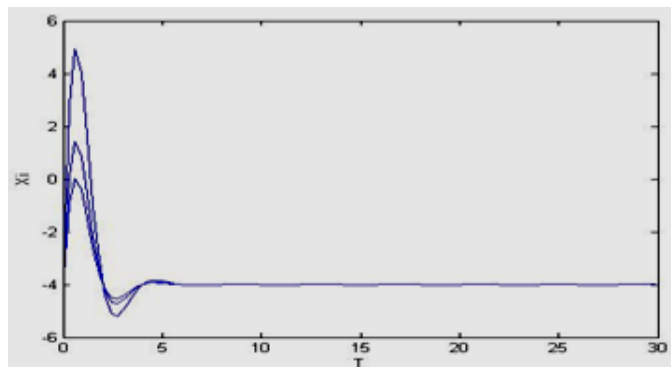


Figure 4.1 The image of xi changing with t in Lorenz system

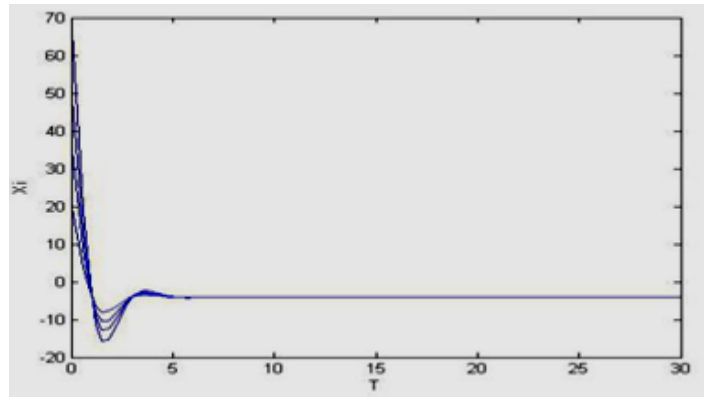


Figure 4.2 The image of xi changing with t in Rossler system

Through some kind of linearized processing on typical Lorenz chaotic system with differential geometric method, the portrait undulating of xi is becoming weakened in figure 4.1, thus the system is stabilized.

From figure 4.2, we can see that, using differential geometric method, the fluctuating in

Rossler system is very stable and has been greatly weakened.

In the stimulation experiment of the steering vehicle, we need to set corresponding parameters for the steering vehicle modeling

Table 1 Parameters of the steering vehicle

Symbol of parameter	Parameter name	Simulation parameter
d	Half of the distance between the left and right wheel	0.64m
M	Gross mass of the vehicle	1030kg
M_s	Mass above the spring	910kg
M_{c1}, M_{c2}	Mass below the spring of wheel 1 and wheel 2	26.5kg
M_{c3}, M_{c4}	Mass below the spring of wheel3 and wheel 4	24.4kg
$k_{t1}, k_{t2}, k_{t3}, k_{t4}$	Rigidity of the tire	138kN/m

We conduct stimulation from two aspects (intersection angle of the fore wheel and vehicle speed) to observe their influence on the tilting of the vehicle. The larger the tilting angle, the greater the corresponding influence. We apply differential geometric method here as well. When the intersection angle of the fore wheel and vehicle speed reach a certain value, the

tilting tendency of the vehicle is controllable within a certain range.

The initial condition of the experiment: (1) the initial speed of the vehicle is 25m/s, set the intersection angle of the fore wheel as 2° , 4.2° , and 6.3° . (2)When the intersection angle of the fore wheel is 5° , the vehicle speed turns to 40m/s from 10m/s

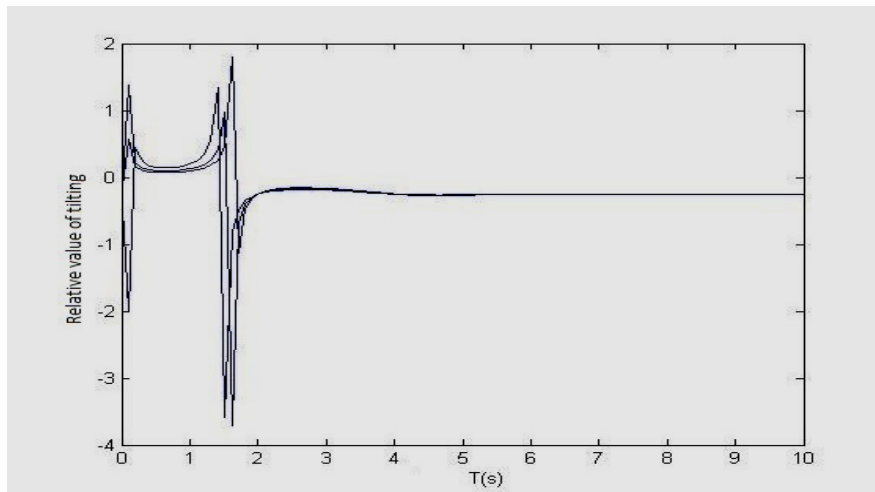


Figure 4.3 Influence of different intersection angle of the fore wheel on tilting

In figure 4.3, we can see that under a certain vehicle speed, the larger the intersection angle of the fore wheel, the greater its influence on tilting of the vehicle body. The active

suspension constructed with differential geometric method makes the relative value of the tilting angle (stable state) -0.23 when the intersection angle of the fore wheel is 6.3° .

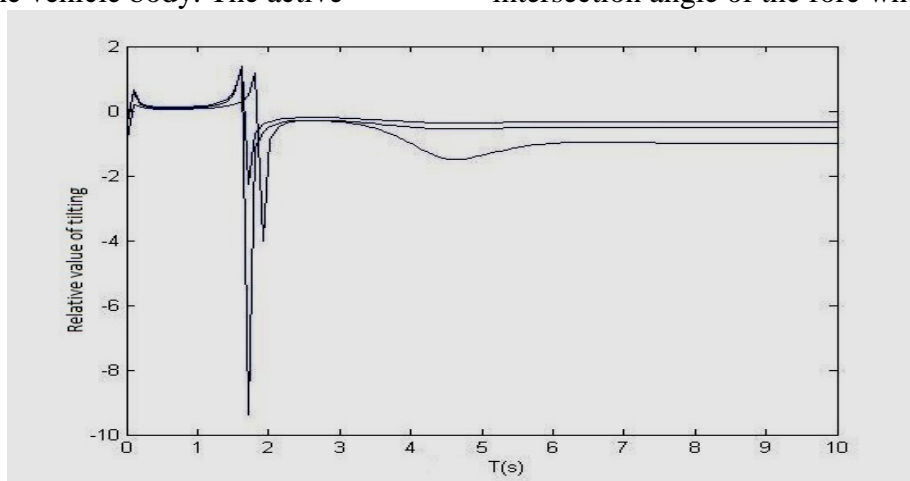


Figure 4.4 influence of different vehicle speed on tilting

In figure 4.4, under the condition of a constant intersection angle of the fore wheel, the larger the initial speed, the less stable the

vehicle body. We apply the relative method of differential geometry, the influence on the body tilting is controllable.

5 CONCLUSION

From different perspectives of the control system, we apply differential geometric methods to promote the systematic stability. In

the chaotic system, we take Lorenz and Rossler as examples, design controller and add it to the initial chaotic equation. Through transformation of coordinates, we make the system linear and controllable. From the perspective of vehicle steering, we apply differential geometric methods to stimulate the process of steering, and also design decoupling control algorithm to weaken the vibration owing to the steering of the vehicle body. From the perspective of UAV guidance modeling, concerning with matrix $SE(3)$ and some mechanics knowledge, we build the motion model and mechanics model of UAV with Newtonian forms.

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