GRAPHICAL METHOD FOR PROFILING THE TOOLS WHICH GENERATE INTERNAL SURFACES BY ROLLING

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ABSTRACT: In this paper, we present a graphical method, developed in CATIA environment, based on the method of substituting circles family and dedicated to profile pinion cutters for generating interior surfaces. Hereby, the family of substituting circles associated to the worked piece (the profile to be generated) is determined at the same time to the curve transposed, in the rolling motion, to the tool centrode. After finding the position of the contact points between the generated profile and the family circles, we can determine, in the transposed family, the locus of the points from the reciprocal enwrapping profile – the pinion cutter profile. The paper also includes two method applications, for generating a hole with square transversal section, and a pump stator with trapezoidal profile. Comparisons between the profiling results when using the graphical method, versus an analytical method have also been realized. The results are proving the coincidence between the two profiles determined as above, on one hand, and both rapidity and precision of the graphical method, on the other hand.
KEY WORDS: graphical method, CATIA, profiled holes, pinion cutter.

1 INTRODUCTION

The profiled hole definition, accepted here, is: cylindrical surface having the transversal section formed by an ordinate whirl of profiles. This type of surfaces have technical application in detachable clutches construction (holes with square or hexagonal sections, slots with parallel flanks or triangular). They can be also met at the stators of special pumps (z/z+1 pumps, gear rims with interior teeth). The generation of such surfaces can be done by milling with machine templates, in the case of a square hole (Berbinschi et al, 2014), by broaching with profile shaped broaches – if the required broach dimension is satisfactory from economical point of view, or by slotting with pinion cutters (Oancea, 2004).

Profiling the pinion cutter used to generate an ordinate whirl of profiles, which constitutes a hole generatrix, can be realized by appealing to fundamental theorems of the enwrapped surfaces by rolling – GOHMAN (Litvin, 1989), (Radzevich, 2007) or to complementary analytical methods, such as: “the minimum distance method”, “the family of substituting circles method” (Oancea, 2004), or “the plain trajectories method” (Teodor, 2010).

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In the present paper we suggest a solution for this type of problem, based on the family of substituting circles method, and developed in a graphical environment – CATIA. This problematic has been already approached for the cases of generating with a rack-tool or with a pinion cutter (but for exterior whirls of profiles) (Ivanov et al., 1998), (Berbinschi et al., 2012, 2013). The graphical method applies the facilities of CATIA graphical environment, which enables to give a rapid and rigorous solution for profiling the pinion cutter used to generate interior surfaces. The qualities and the accuracy of the new-developed method are assessed by comparison to the known analytical methods.

2 THE “FAMILY OF SUBSTITUTING CIRCLES” METHOD

We further present the family of substituting circles method, applied in the particular case of enwrapping between profiles associated to a couple of interior rolling centrodes (Fig. 1). The following reference systems are considered:

- xy, meaning a global system, attached to C_1 centrode axis;
- x_0y_0 – global system, solidary to the rotation axis of C_2 centrode;
- XY – relative system, attached to C_Σ profile to be generated and also to C_1;
- $\xi\eta$ – relative system, solidary to C_2 centrode of the pinion cutter.

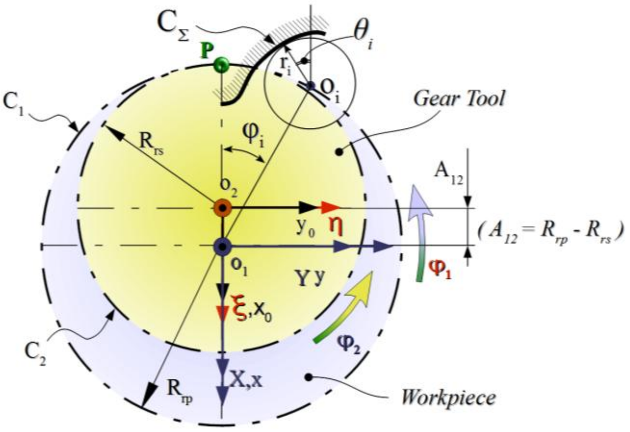

Figure 1. Interior tangent centrodes; the family of substituting circles

The family of substituting circles equations (see Fig. 1), referred to *XY* system, are:

$$\left(C_i\right)_{\varphi_1} \left| \begin{array}{l} X = -R_{rp} \cdot \cos\varphi_1 - r_i \cdot \cos\theta_i\,; \\ Y = R_{rp} \cdot \sin\varphi_1 - r_i \cdot \sin\theta_i\,, \end{array} \right. \tag{1}$$

with φ_i and θ_i angular variables and r_i – the radius of current circle from the substituting family.

The family of substituting circles $\left(C_i\right)_{\varphi_1}$ is a family of circles having their centers on C_1 centrode and being tangent to C_Σ profile:

$$C_\Sigma \left| \begin{array}{l} X = X(u); \\ Y = Y(u), \end{array} \right. \tag{2}$$

where u means an arbitrary variable.

The two centrodes C_1 and C_2 are in rolling motion (see Fig. 1), and obey to the following condition:

$$R_{rp} \cdot \varphi_1 = R_{rs} \cdot \varphi_2\,,\ \text{or}\ \varphi_2 = i \cdot \varphi_1\,, \tag{3}$$

i meaning the transmission ratio (usually, a constant parameter). The expression of the distance between the centers of the two centrodes is:

$$a = R_{rp} - R_{rs}\,. \tag{4}$$

The following relations issue from the condition of tangency between the substituting circles family (1) and C_Σ profile (2):

$$\begin{cases} -R_{rp} \cdot \cos\varphi_1 - r_i \cdot \cos\theta_i = X(u); \\ R_{rp} \cdot \sin\varphi_1 - r_i \cdot \sin\theta_i = Y(u); \\ r_i \cdot \sin\theta_i = X'(u); \\ -r_i \cdot \cos\theta_i = Y'(u). \end{cases} \tag{5}$$

The system (5) enables to find, for a given value of φ_1 parameter, the magnitudes of r_i and θ_i – the current circle radius and the position, on this circle, of the tangency point to C_Σ profile.

According to "family of substituting circles" method, the envelop of C_Σ profile, in the rolling motion between the two centrodes, is defined by the transposed of the circles family to C_2 centrode, associated to the tool, in the relative motion:

$$\xi = \omega_3(\varphi_2)\left[\omega_3^T(\varphi_1) \cdot X - A\right], \tag{6}$$

with

$$A = \begin{pmatrix} -a \\ 0 \end{pmatrix}, \tag{7}$$

where *X* matrix means the circles family (1), with r_i and θ_i given by conditions (5), see also (4).

3 GRAPHICAL METHOD

We further present a graphical method, developed in CATIA environment for profiling the pinion cutter type of tool, used to generate an ordinate whirl of profiles, defining an ensemble of interior surfaces (for example polygonal holes, interior slots, interior gear rims with involute profile teeth, pump stators).

Fig. 2 illustrates the principle of determining the substituting circles family for C_Σ profile, based on the following succession of steps:

- For the section delimited by the characteristic points *A* and *B*, the normal to profile are drawn in these points. Their intersection with the circle of R_{rp} radius determines the points *a* and *b* (see Fig. 2), corresponding to an angle at center denoted by δ.
- The angle δ is divided in n equal parts, or the co-ordinates of $O_{i,p}$ points are arbitrary chosen on the circle of R_{rp} radius, as well.

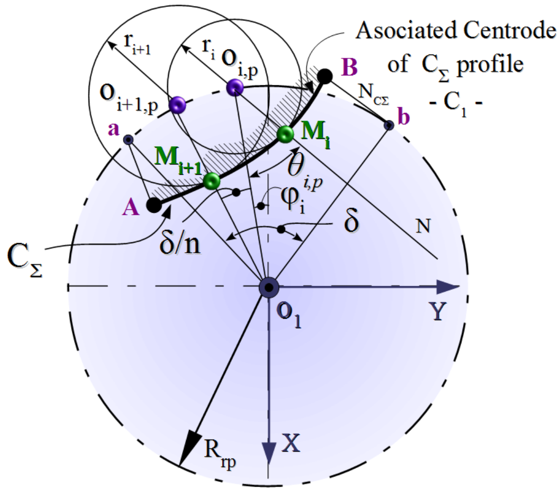

Figure 2. The substituting circles family determined in CATIA environment

- The circles tangent to C_Σ profile, and having the centers in the $O_{i,p}$ points, are drawn (PROJECT and CIRCLE commands).
- The $\theta_{i,p}$ angles, formed between $O_1O_{i,p}$ radius and the normal in $O_{i,p}$ to C_Σ profile are measured.

The co-ordinates of M_i points – the tangency point between the circle $O_{i,p}$ and C_Σ profile are determined (PROJECT command).

These graphical procedures are repeated for all the circles from C_Σ profile substituting family, and then the circles family is transposed on the tool centrode, as it follows (see also Fig. 3).

- The $O_{i,s}$ points, owning to the circle of R_{rs} radius, associated to the pinion cutter, are determined. They are corresponding to $O_{i,p}$ points from C_1 centrode, in the rolling motion, see also relation (3). The angle between two successive points, $O_{i,s}$ and $O_{i+1,s}$ is $i{\cdot}\delta/n$.
- The circles of r_i radius and having their centers in $O_{i,s}$ points are drawn (CIRCLE command).
- The straight line N, forming the angle $\theta_{i,p}$ with $O_2O_{i,s}$ direction, is drawn – see Fig. 2; at its intersection with the circle of r_i radius we have the point $M_{i,s}$. The totality of $M_{i,s}$ points, determined as above, are giving the pinion cutter profile, S, reciprocal enwrapped to C_Σ profile, in the rolling process between the two centrodes C_1 and C_2.

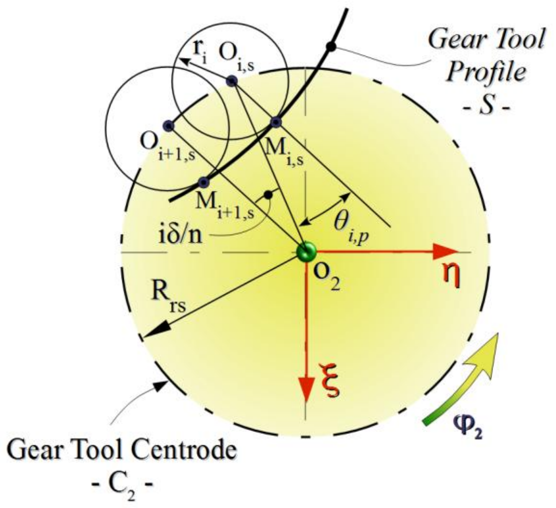

Figure 3. The substituting circles family transposed on the tool centrode

4 NUMERICAL APPLICATIONS

We now exemplify the application of the graphical method in two different cases, for profiling the pinion cutter used to generate a hole with square transversal section and a pump stator with trapezoidal profile.

4.1 The pinion cutter for generating a square hole

Fig. 4 illustrates the graphical method application in order to solve the tool-profiling problem when generating a square hole with a pinion cutter. The needed reference systems and the geometrical parameters, which have to be considered are here presented.

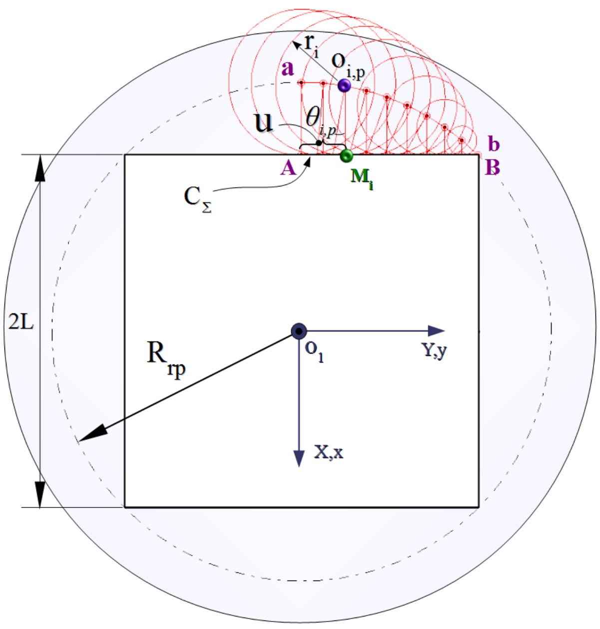

Figure 4. The square section hole

The radius R_{rp} of the centrode associated to the generated profile has to satisfy the condition that the normal, considered in any of the profile points, intersects the circle of R_{rp} radius. Due to the symmetry, it is sufficient to solve the problem for the *AB* segment (Fig. 4).

In the case of the analytical solution, the profile C_Σ, *[AB]* segment, is given through the equations:

$$\left| \begin{array}{l} X = -L; \\ Y = u, \end{array} \right. \tag{8}$$

with u variable respecting the restrictions $0 \le u \le L$. The condition to find the family of substituting circles leads to the following relations (see also Fig. 4):

$$\begin{array}{l} \sin\varphi_1 = \dfrac{u}{R_{rp}}; \quad \theta_i = 0; \\ r_i = -L + \sqrt{R_{rp}{}^2 - u^2}; \quad R_{rp} = \sqrt{2}\cdot L, \end{array} \tag{9}$$

which, when *u* remains inside the limits above specified, determines the substituting circles family.

According to the algorithm presented in the previous section, the family of circles is also found with the help of CATIA environment graphical facilities (see Fig. 5), when L = 20 mm, and R_{rp}/R_{rs} = 4/3.

The pinion cutter profile, as obtained by both analytical and graphical methods, is represented in Fig. 6. At the same time, in Table 1 are comparatively exemplified the co-ordinates of some among the points determining the tool profile, as they resulted from each of the two methods application.

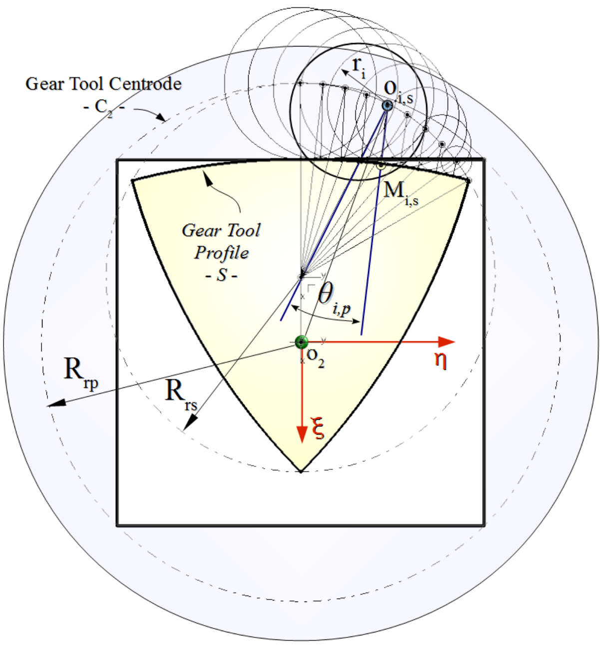

Figure 5. The substituting circles family – the transposed on the tool centrode

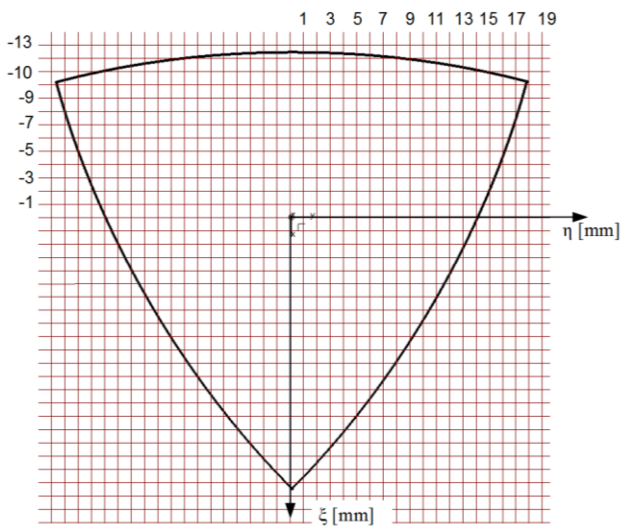

Figure 6. The pinion cutter profile

Table 1. The co-ordinates of points from tool profile

Crt. no.	Analytical method		Graphical method	
	ξ [mm]	η [mm]	ξ [mm]	η [mm]
1	-12.929	0.000	-12.929	0.000
2	-12.898	2.166	-12.897	2.225
3	-12.806	4.335	-12.800	4.435
4	-12.635	6.689	-12.641	6.617
5	-12.431	8.690	-12.423	8.757
6	-12.142	10.883	-12.149	10.840
7	-11.814	12.909	-11.824	12.855
8	-11.458	14.767	-11.454	14.789
9	-11.041	16.649	-11.046	16.631
10	-10.607	18.371	-10.607	18.371

As it can be noticed, by analyzing the results in graphical form, the profiles determined by using the two mentioned methods are coincident, even if between the co-ordinates from Table 1 there are some differences. They are due to the fact that, excepting the extremities, we are not talking about exactly the same points along the profile – because of its specific, each of the methods gives the same profile, but through different series of points.

4.2 The pinion cutter for generating a pump stator with trapezoidal profile

The pumps with stator having rectilinear-profile teeth are used for operating viscous fluids in drugs or cosmetics industry as well as in factories producing building materials. Rectilinear segments and arcs of circle compose the profile of such a stator tooth (Fig. 7).

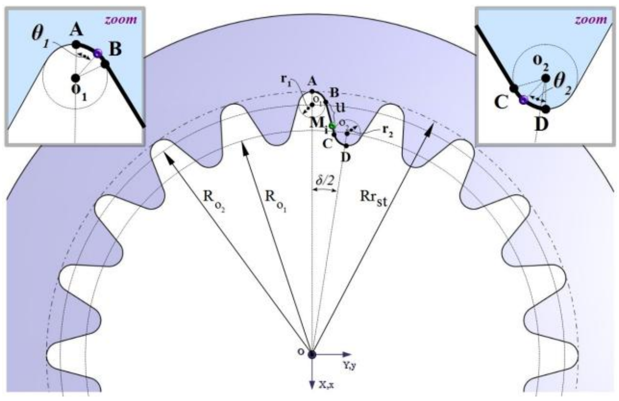

Figure 7. The pump stator profile

Thereby, the stator tooth profile from Fig. 7, referred to XY system is formed by the following simple curves:

- The arc of circle AB of r_1 radius, having the equations

$$AB\left|\begin{array}{l} X=-R_{O_1}-r_1\cos\theta_1; \\ Y=-r_1\sin\theta_1, \end{array}\right. \qquad (10)$$

with θ_1 variable parameter.

- The rectilinear segment *BC*, of equations

$$BC\left|\begin{aligned}&X=-R_{O_1}-r_1\sin\beta+u\cos\beta;\\&Y=-r_1\cos\beta-u\sin\beta,\end{aligned}\right. \quad (11)$$

with u variable parameter.

- The arc of circle CD, of equations

$$CD\left|\begin{aligned}&X=-R_{O_2}\cos\left(\frac{\delta}{2}\right)-r_2\cos\left(\theta_2+\frac{\delta}{2}\right);\\&Y=-R_{O_2}\sin\left(\frac{\delta}{2}\right)+r_2\sin\left(\theta_2+\frac{\delta}{2}\right),\end{aligned}\right. \quad (12)$$

with θ_2 independent variable.

The limits of variation for variable parameters from equations (10) – (12) can be established according to constructive reasons, as it follows:

- For the angular parameters, we have

$$\begin{cases}\theta_{1\min}=0;\\ \theta_{1\max}=\dfrac{\pi}{2}-\beta,\end{cases} \quad (13)$$

respectively

$$\begin{cases}\theta_{2\min}=0;\\ \theta_{2\max}=\dfrac{\pi}{2}-\beta.\end{cases} \quad (14)$$

- For finding the *u* parameter limits, we firstly determine, with equations (10) and (12) associated to conditions (13) and (14), the co-ordinates of points *B* and *C*,

$$\begin{cases}X_B=-R_{O_1}-r_1\sin\beta;\\ Y_B=-r_1\cos\beta;\end{cases} \quad (15)$$

$$\begin{cases}X_C=-R_{O_2}\cos\left(\dfrac{\delta}{2}\right)-r_2\sin\left(\beta-\dfrac{\delta}{2}\right);\\ Y_C=-R_{O_2}\sin\left(\dfrac{\delta}{2}\right)+r_2\cos\left(\beta-\dfrac{\delta}{2}\right).\end{cases} \quad (16)$$

Then, we can find

$$\begin{cases}u_{\min}=0;\\ u_{\max}=\sqrt{(X_B-X_C)^2+(Y_B-Y_C)^2}.\end{cases} \quad (17)$$

In Fig. 8 there are presented the two rolling centrodes, C_1 and C_2, having the radii R_{rst} and R_{rsc}, respectively; they must obey the rolling condition:

$$R_{r_{st}}\cdot\varphi_1=R_{r_{sc}}\cdot\varphi_2. \quad (18)$$

The following reference systems are defined:

- *xy* and x_0y_0, as absolute systems;
- *XY* – mobile, attached to the stator profile;
- $\xi\eta$ – mobile, attached to the pinion cutter.

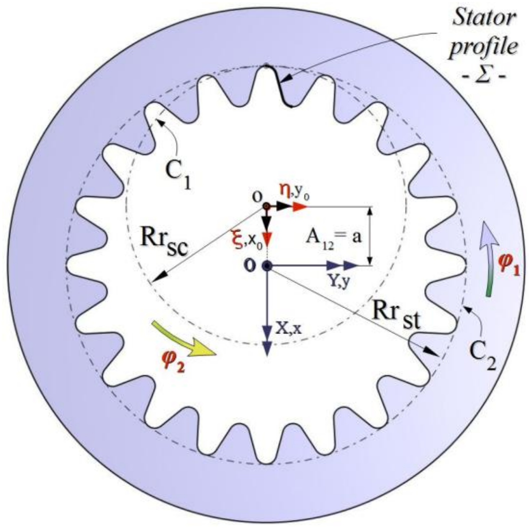

Figure 8. The couple of rolling centrodes

The motion of *XY* system relative to $\xi\eta$ system, described by (6), enables us to find, after calculus, the family of Σ profiles in the tool reference system. From this point on, the application of one among the already known methods to determine this family' envelop leads to the solution meaning the pinion cutter profile.

However, if we intend to solve the problem by using the graphical method, then a substituting circles family is firstly associated to the frontal profile of the pump stator (Fig. 9). According to the suggested profiling algorithm, the transposed, the transposed of this family on the tool centrode, of R_{rsc} radius is then determined (Fig. 10). The ensemble of C_i circles tangency points with the three curve segments – AB, BC and CD, along the rolling motion, gives the pinion cutter profile.

The Fig. 11 presents the tool profile obtained for the stator tooth profile characterized by $r_1 = r_2 =$ 3 mm, $\delta/2 = 30°$, $R_{O1} = 55$ mm and $R_{O2} = 45$ mm.

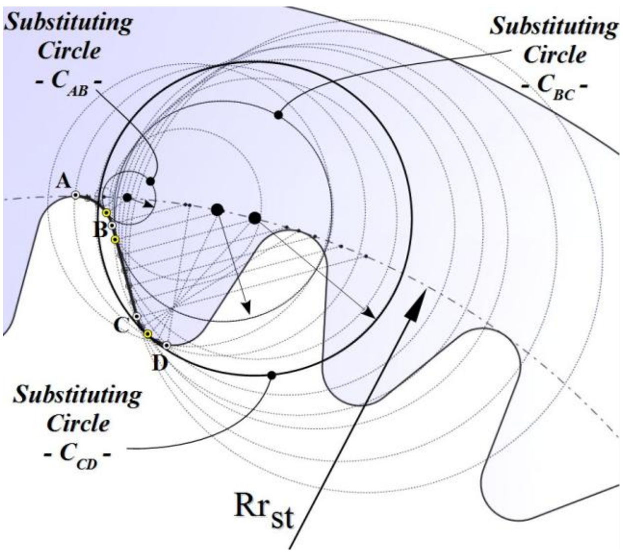

Figure 9. The substituting circles family

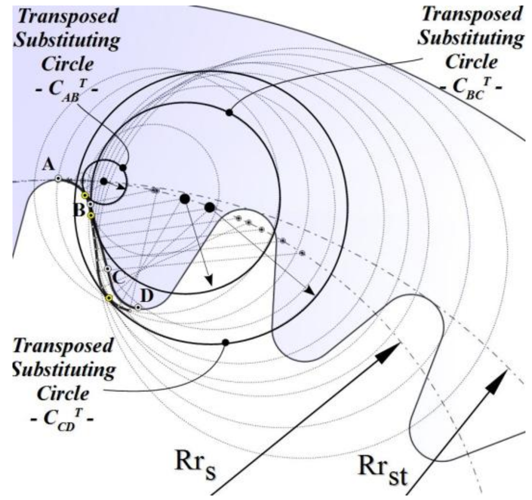

Figure 10. Transposed of substituting circles family

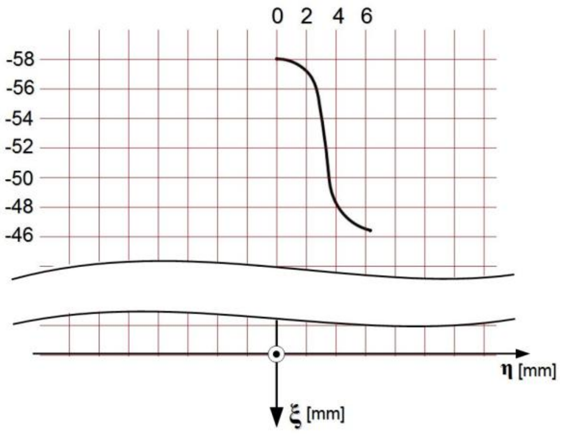

Figure 11. The pinion cutter profile

Table 2. Points co-ordinates

	ξ [mm]	η [mm]
AB sector	0.0000	-58.0000
	0.5395	-57.9509
	1.0602	-57.8012
	1.5443	-57.5579
	1.9777	-57.2328
	....	
BC sector	....	
	3.2528	-53.0772
	3.3319	-52.5078
	3.4054	-51.9377
	3.4735	-51.3668
	3.5362	-50.7954
	....	
CD sector	....	
	4.6641	-47.1317
	5.0596	-46.7461
	5.5095	-46.4256
	6.0040	-46.1795
	6.5309	-46.0139

In Table 2 there are also exemplified the co-ordinates of some among the points from the tool profile, found by applying the graphical method.

5 CONCLUSION

The graphical method suggested in this paper turns to profit the facilities of CATIA graphical environment. The solution offered by the graphical method is rigorous (this being proven by comparison with analytical methods), rapid, and, at the same time, intuitive. The method is also appropriate for studying complex interior surfaces and has the important advantage of diminishing the risk of giving a wrong interpretation to numerical data regarding a pinion cutter profiling.

6 ACKNOWLEDGEMENTS

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