CONTACT RATIO CALCULATION OF INVOLUTE ARC GEAR DEVICE

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ABSTRACT: Following the basic principles of gear meshing and the computing theories of gear contact ratio, this paper deducts the contact ratio formula of involute arc gear device (IAGD), and analyzes the effect of the ratio. The proposed formula reveals that the IAGD contact ratio is positively correlated with the circular arc curvature and negatively with pressure angle. The research results shed new light on the theory and application of the IAGD design.

KEY WORDS: involute arc gear device (IAGD), contact ratio.

1 INTRODUCTION

As one of the most popular options for gear mechanism, the cylindrical gear device mainly consists of spur gears, helical gears and herringbone gears. To overcome the structural defects of these gears, the arc gear has been developed (Koga, 1975) to achieve the transmission of ordinary cylindrical gear with an improved structure. The new type of gear has attracted the attention of numerous theorists (Tseng, 2004; Dai, 2006; Kou, 2002; Song, 2006; Ma, 2005; Faydor and Litvin, 1994).

Defined as the ratio of the arc of action to the pitch circle, the contact ratio is a structural indicator of gear pair that reflects the continuity, smoothness and uniformity of the transmission load of the gear pair, and illustrates the load distribution between the meshing teeth of the pair. The ratio has a direct bearing on various parameters, such as the noise control of gear transmission, the bending strength of tooth root, the contact strength of tooth surface, the adhesion of tooth surface, etc.(Faydor and Litvin,1994)

Following the basic principles of gear meshing and the computing theories of gear contact ratio, this paper deducts the contact ratio formula of involute arc gear device (IAGD), and analyzes the effect of the ratio.

2 GENERATION OF IAGD TOOTH SURFACE

As shown in Figure 1, chord *CD* of circular arc *CGD* on generating plane *S* is parallel to busbar *NN'* of the base cylinder. The radius and center of the arc are R_t and Q_t, respectively.

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As surface *S* rotates along the base cylinder, the $C_0G_0D_0$ becomes the curve of the circular arc *CGD* on the cylindrical surface, turning the trajectory of the

point on *CGD* into the tooth surface of the IAGD (Jiang, 2014).

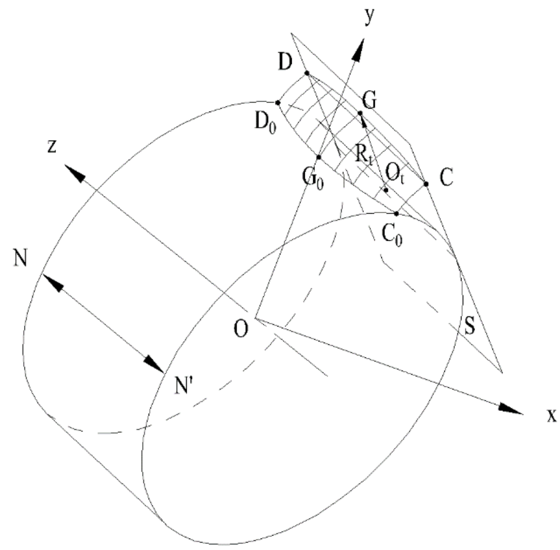

Figure 1. Generation of the tooth surface of the IAGD

Assuming that the axis of the base cylinder is axis *z*, the middle radial cross section is coordinate plane *xoy*, and midpoint G_0 of circular arc $C_0G_0D_0$ is on axis *y*, the coordinate system $S(x, y, z)$ can be established on the frame of reference, and the equation of tooth surface in the coordinate system can be acquired as below (Jiang, 2014):

$$\begin{cases} x = r_{b1}(\sin\theta_t - \phi\cos\theta_t) \\ y = r_{b1}(\cos\theta_t + \phi\sin\theta_t) \\ z = h \end{cases} \tag{1}$$

where r_{b1} is the radius of the base cylinder; θ_t is the turning angle of the generating plane S; ϕ is the opening angle of random point M on CGD when S is rotated.

3 IAGD CONTACT RATIO

3.1 Endface contact ratio of the IAGD

If the IAGD is radially sectioned into infinitely narrow pinions, then each pinion can be approximated as an involute spur gear. The contact ratio of the pinion end face, which is in the shape of involute tooth profile, can be calculated in the same way as that of involute spur gear. Here, the contact ratio is denoted as ε_α. Figure 2 shows the tooth profile β-β and γ-γ of the teeth O_1 and O_2 in three different positions.

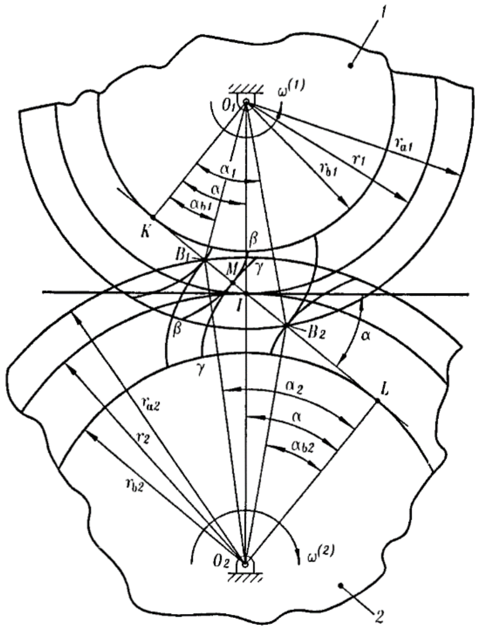

Figure 2. Random radial crosssection of involute gear meshing

Hereto, it is possible to examine the rotating angle of the teeth pair of two tooth profiles in one meshing cycle from the beginning to the end of contact. Suppose the pinion angle and the gear angle are $\angle B_1O_1B_2$ and $\angle B_1O_2B_2$, respectively, the tangential contact of two adjacent tooth profiles is a continuous process under the following conditions

$$\angle B_1O_1B_2 \geq \frac{2\pi}{z_1} \qquad \angle B_1O_2B_2 \geq \frac{2\pi}{z_2} \qquad (2)$$

where z_1 and z_2 are the teeth number of gears O_1 and O_2, respectively.

Thus, the contact ratio can be expressed as:

$$\varepsilon_\alpha = \frac{l}{P_b} = \frac{l}{p_c \cos a} \qquad (3)$$

where $l = \overline{B_1B_2}$ is the length of the working part of the meshing line, that is, the contact point displacement along the meshing line in the gear cycle; P_b is the measured distance between the two adjacent tooth profiles in the common line direction; P_c is the distance between the two teeth on the pitch circle; a is the pressure angle of the gear.

The following equation holds in accordance with Figure 2:

$$\overline{KB_2} + \overline{B_1L} = \overline{KL} + l \qquad (4)$$

Therefore,

$$l = r_{b1}\tan a_{a1} + r_{b2}\tan a_{a2} - (r_{b1} + r_{b2})\tan a \qquad (5)$$

where r_{ai} and r_{bi} are the base radiuses of the gear pair, respectively.

3.2 Axial direction contact ratio

For the involute spur gear, the mating process develops along the pitch circle (Figure 3(a)). The gear teeth are meshed across the entire tooth width both during the engagement at A_1A_1 and the disengagement at A_2A_2. Hence, the contact ratio of the gear equals l/P_b, where l is the engaging length and P_b is the end face distance of the normal tooth.

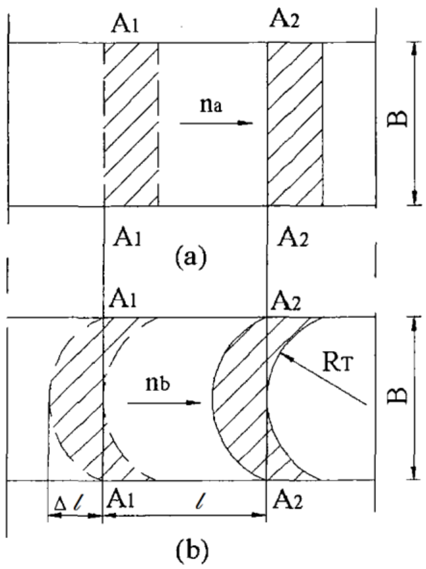

Figure 3. Contact ratio calculation of the IAGD

For the IAGD, the meshing process develops along the pitch circle (Figure 3(b)). During the engagement at circular arc A_1A_1, the middle part of A_1A_1 meshes earlier than the two ends; during the disengagement at circular arc A_2A_2, the middle part of A_1A_1 disengage earlier than the two ends.

According to in Figure 3 (b), the actual meshing length of the IAGD is Δl longer than that of the involute spur gear. Therefore, the IAGD has a greater contact ratio than the involute spur gear.

The gear circumferential angle subtended by Δl equals the circumferential angle corresponding to the arc teeth in Figure 1, i.e. the circumferential angle θ_n subtended by the distance of point G to chord CD.

$$\theta_n = (R_t - \sqrt{R_t^2 - (B/2)^2}) / r_b \quad (6)$$

Owing to the dependence on tooth width B and circular arc tooth radius R_t, the contact ratio corresponding to Δl is called the axial contact ratio ε_β, which is expressed as:

$$\varepsilon_\beta = \frac{\Delta l}{p_b} = \frac{(R_t - \sqrt{R_t^2 - (B/2)^2})\cos\alpha}{p_b} = \frac{(R_t - \sqrt{R_t^2 - (B/2)^2})}{\pi m_c}$$

(7)

where α is the standard pressure angle on the end face of the gear; m_c is the axial modulus; B is the tooth width.

3.3 IAGD contact ratio

The IAGD contact ratio ε_r is the sum of the end face contact ratio and the axial contact ratio:

$$\varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta$$

(8)

The expression of ε_r is obtained by substituting formulas (3) and (7) into formula (8):

$$\varepsilon_\gamma = \frac{Pl}{\pi \cos a} + \frac{(R_t - \sqrt{R_t^2 - (B/2)^2})}{\pi m_c}$$

(9)

4 INFLUENCING FACTORS OF CONTACT RATIO

It can be seen from formula (9) that arc radius R_t and pressure angle a are directly correlated with the IAGD contact ratio. The two parameters both affect the intuitive geometry of the gear, and determine the shape of the gear tooth profile. Thus, it is very meaningful to explore the relationship between R_t, a and the contact ratio. Substituting formula (5) into formula (9), we can acquire the following formula:

$$\varepsilon_\gamma = \frac{r_{b1}\tan a_{a1} + r_{b2}\tan a_{a2} - (r_{b1}+r_{b2})\tan a}{p_c \cos a} + \frac{(R_t - \sqrt{R_t^2 - (B/2)^2})}{\pi m_c}$$

(10)

4.1 Effect of pressure angle on the contact ratio

According to formula (10), the derivative of a is obtained as:

$$\frac{\partial \varepsilon_\gamma}{\partial \alpha} = -\frac{r_{b1}\tan a_{a1} + r_{b2}\tan a_{a2} - (r_{b1}+r_{b2})\tan a}{p_c \cos^2 a} \sin a - \frac{(r_{b1}+r_{b2})}{p_c \cos^3 a}$$

(11)

As shown in Figure 2,

$r_{b1}\tan a_{a1} + r_{b2}\tan a_{a2} - (r_{b1}+r_{b2})\tan a \geq 0$ and $0 \leq \alpha \leq \pi/2$. Thus, it is concluded that $\frac{\partial \varepsilon_\gamma}{\partial \alpha} \leq 0$, indicating that the IAGD contact ratio decreases as the pressure angle a increases. In other words, a low pressure angle is favorable for the transmission of the gear device. However, the pressure angle should not be too small, for the chance of rooting cutting grows with the decline in the angle.

4.2 Effect of arc curvature on the contact ratio

Similarly, the derivative of arc radius R_t is acquired as:

$$\frac{\partial \varepsilon_\gamma}{\partial R_t} = \frac{\sqrt{R_t^2 - (B/2)^2} - R_t}{\pi m_c \sqrt{R_t^2 - (B/2)^2}}$$

(12)

Since $B/2 \leq R_t$, it is concluded that $\partial\varepsilon_r/\partial R_t \leq 0$. Obviously, the contact ratio grows with the radius R_t of the circular arc.

5 CONCLUDING REMARKS

In this part will be emphasize the contributions of the paper and the future applications in the research field.

(1) According to the principles of gear meshing, the contact ratio formula of the IAGD was deduced, laying the theoretical basis for the design of pressure angle a and arc radius R_t of the involute gear.

(2) The IAGD contact ratio is positively correlated with the circular arc curvature and negatively with pressure angle.

(3) During the parameter design of the IAGD, the proper selection of pressure angle and arc curvature could increase the contact ratio, and thereby ensure the continuous and smooth operation of the gear.

6 REFERENCES

►Dai, Y.T., Ariga, Y., Jiang, D.S. (2006). Hobbing mechanism of cylindrical gear with arcuate tooth traces and experimental investigation. China Mechanical Engineering, 17(7): 706-709.

►Faydor, L.L. (2014). Gear Geometry and Applied Theory. New Jersey: PTR Prentice Hall, 316.

►Jiang, Y.Q., Hou, L. (2014). Meshing features of involute arc teeth cylindrical gears. Journal of Chemical and Pharmaceutical Research, 6(7): 2387-2393.

►Jiang, Y.Q., Hou, L., Zhao, Y. (2014). The Equation of the Contact Line of the Involute Curvilinear-Tooth Cylindrical Gear Pump for the Agricultural Tractor. The Open Mechanical Engineering Journal, 8: 879-884.

►Koga, T. (1975). Method for cutting paired gears having arcuate tooth trace:American, 3915060.

►Kou, S.Y., Liu M.B., Wu L.C. (2002). The new process method of arc cylindrical gear. Machinery Manufacturing Engineer, 9: 16-18.

►Ma, Z.Q., Wang, X.C., Shen B. (2005). Real Tooth Contact Analysis of the clindrical Gears with etrical Arcuate Tooth Trace. Journal Of Xi'an Jiaotong University, 39(7): 722-725.

►Song, A.P., Yi H., Tang, W.C. (2006). Involute arc cylindrical gear and its mesh characteristics. China Mechanical Engineering, 17(18): 1888-1892.

►Tseng, R.T., Tsay, H.B. (2004). Contact characteristics of cylindrical gears with curvilinear shaped teeth. Computer Methods in Applied Mechanics and Engineering, 9: 905-919.