RESEARCH ON THE MANUFACTURING OF MECHANICAL PARTS BASED ON THE THEORY OF SPACE SYMMETRY GROUP

Xiaolei DENG[1]*, Jianchen WANG[2]

ABSTRACT: The symmetric structure of parts will affect the manufacturing, and the assembly time is an important factor in influencing the manufacturing of mechanical parts. For more comprehensive study of the symmetry of parts and relations with machine parts assembly ability, this paper puts forward the concept of mechanical structure symmetry, analyzes the factors influencing the structural symmetry, and establishes the single body structure symmetric calculation model by the symmetrical structure decomposition diagram. In addition, the paper also puts forward the concept of parts total correction angle, establishes a "symmetry - assembly time" prediction model suitable for the parts in single body and multi-body cases, and through the cases, analyzes the effect of symmetry on the assembly of mechanical parts. The results showed that the symmetry of parts can make an influence on the assembly time of mechanical parts. For single symmetrical parts and single-body multiple-symmetrical parts, the higher the parts symmetrical degree, the less time-consuming for assembly, the higher the assembly efficiency, and the higher the assembly ability. If the symmetry difference of symmetric directional and body silhouette is very small, it should expand the differences between them, so as to facilitate the human or machine identification.

KEY WORDS: mechanical parts; assembly time; manufacturing.

1 INTRODUCTION

Symmetry group theory is an important mathematical tool for describing the symmetry of an object. It has been profoundly studied in many fields, such as crystal, molecule and quantum mechanics. However, in the field of machinery, although mechanical structure symmetry is the problem often encountered in mechanical design, in mechanical products, there are many symmetrical structures. Whereas, there is no systematical understanding of the effect of using symmetrical structure on the manufacturing of mechanical parts. In the field of machinery, the existing research on symmetry group theory is mainly about the study of computer aided symmetry identification and matching of mechanical structure, not systematically focusing on the symmetry group of mechanical parts (Zhou, W.J.et al., 2016).This paper introduced the concept of symmetry group in natural science into the field of mechanical engineering for the first time, and with symmetry group theory, put forward the concept of mechanical structure symmetry. Moreover, it analyzed the factors affecting the structure symmetry, established the single main structure symmetric calculation model through the structural symmetry decomposition diagram, and analyzed the influence of symmetry of mechanical parts on the assembly time in the mechanical parts manufacturing process.

[1] College of Mechanical Engineering Quzhou University, Quzhou, Zhejiang 324000,China

E-mail:xiaoleideng_cmequ@126.com.

2 RESEARCH CONTENT

1.1 Symmetry group structure of mechanical parts

Symmetrical mechanical parts structure can be naturally divided into space group structure and point group structure. At the same time, by the combination of parts in the structure, it includes symmetric single and symmetric combination parts these two types structure. Symmetrical single part generally refers to the individual parts, while the symmetrical combination part generally refers to the parts combination with symmetrical structure composed of several parts.

(1) Space group structure of mechanical parts

The part structure with the space symmetry group is a kind of lattice structure with infinite repetition of the same units in the strict sense. Generally speaking, the structure of mechanical parts is a kind of limited and closed structure. In the strict sense, it is possible to use the mechanical part premise described by the space symmetry group to make the structure boundary ignored.

In the field of machinery, such as blade, metal mesh, layout of the same shear connectors, and the storage structure of the same parts and so on, after ignoring the boundary, it is an infinitely repeated structure of a same unit, having certain translation symmetry, which can be described as a specific space symmetry group structure.

(2) Point group structure of mechanical parts

Point symmetry group structure is the most common mechanical parts structure form in the mechanical field. Each part we processed is basically absolutely limited closed geometry, and accordingly, the geometric images can be described with point symmetry group.

Mechanical parts structure of specific domain often has a certain point symmetry group orientation. For example, the bending piece of meal gold is often some non-axial group structure; rotary cutting tools are basically finite uni-axial group structure. The gear group has many forms, but in order to ensure the continuous transmission, generally do not use non-shaft or shaft group structure with low rotation. Relatively speaking, the point group structure with likely shaft is relatively rare in the field of mechanical field. For the group structure , the commonly seen symmetrical single parts include staggered tooth three-edge milling cutter and the tire with cross lines, and inserted tooth three face milling cutter belonging to the assembly part and bicycle rim ring with alternate series and so on (Yi, B.et al.,2016), while the cross slider coupling is the typical group structure. At the same time, the mechanical parts structures belonging to the cube group are very rare, and this paper basically not involves in the point group structure. Infinite time axis group and group structures, or called rotary structure, are more commonly seen in the unformed blank before the parts processing, which is generally a symmetry requirement for ensuring processing quality.

(3)Mechanical structure symmetry and its influencing factors

Definition 1: symmetry degree - symmetry of the symmetric structure. The symbol of the symmetry degree is denoted as SD (Symmetry Degree), and the

symmetry is used to compare the difference between different kinds of ideal single symmetry and the multi-symmetry structures in symmetry degree. The larger the SD value is, the more symmetric the structure is.

Influencing factors: geometrically similar structures have the same symmetry degree SD, irrespective of their corresponding size. The symmetry degree SD of specific structure is only determined by its symmetrical body, symmetry group elements, and symmetric benchmark these three elements, namely: (1) the relationship between the number and spatial position of the symmetry subject; (2) dimensions, types and numbers of symmetrical benchmark; (3) the number of symmetrical components.

1.2 Symmetry model of mechanical components with single body structure

Single symmetry can only be rotation, translation or mirror symmetry. The benchmark dimensions, types and numbers of the single symmetry are unique. The number of symmetric components is the only factor that affects the symmetry degree. The symmetry degree SD of a single symmetric structure has a positive

correlation with the number of symmetric components in unique benchmark.

The existing two methods of calculating the symmetry are the group order method and the total symmetry angle method. For all single symmetries, no matter it is groups order or symmetric angle, it showed a linear relationship with the number of symmetrical components number .

(1) Single dimension symmetry

In addition to the single symmetry, the symmetry of multi-category under the single dimension must be composed of three basic symmetry composites or combinations of rotation, mirror and translation. Figure 1 shows that there is a dimension in the Descartes coordinate system (along the EF line). Based on the constraint of this kind of spatial relationship, there are at most three symmetrical benchmarks under EF dimension, respectively ABCD mirror surface benchmark vertical to EF, translation symmetry benchmark along the EF line, and rotational symmetric benchmark along the EF line. Since that it is the single subject symmetry, the number of different types of benchmarks is only 1.

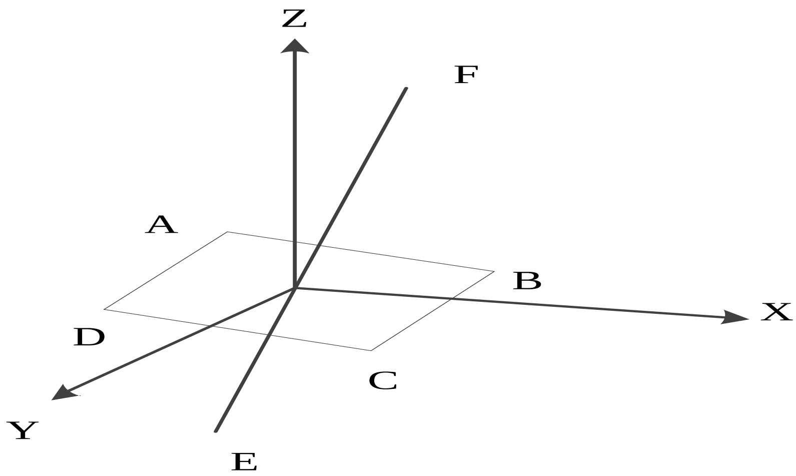

Figure 1. The benchmark types and number under a certain dimension

Rotation transition symmetry ( ) and rotation mirror symmetry ( ) are the typical single dimension multi-type symmetries: the rotation axis of rotation transition symmetry completely coincides with the translation axis, in the same dimension (Yokoki, Y.et al., 2017); the mirror surface of the rotation mirror symmetry is completely vertical to the rotation axis, also in the same dimension. The symmetry degree SD of a single dimension multi-type symmetric structure is jointly determined by the symmetric type number 1 and the number set of components in various classes, which meets the correlation relationship .

(2) Multi-dimension symmetry

Multi-dimension symmetry is the composition or combination of single dimension and multi-type symmetry on multiple dimensions. The symmetry degree SD of multi-dimension symmetric structure is jointly decided by the number m of symmetric dimensions, the symmetric species number set L, and the number set of components of various classes, which meets the following positive correlation relationship .

(3) Symmetry degree modeling

The purpose of the study of the symmetry degree model is to provide a quantitative tool for the comparison of the symmetry degree. For any single subject structure symmetry, we can decompose it from the dimension, type and the number of symmetry components of the benchmark, and get Figure 2.

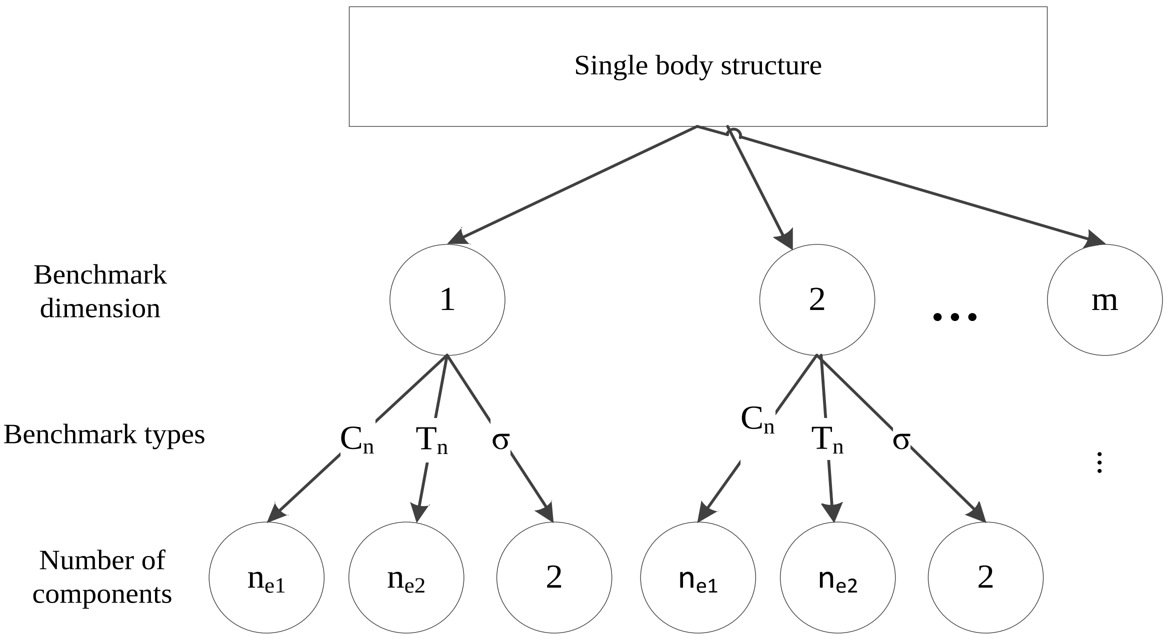

Figure 2. Symmetric decomposition of single body structure

When the benchmark dimension and symmetric components number of single body structure symmetry are limited, we can obtain the structure symmetry degree by (1):

$$SD = \sum_{i=1}^{m} (n_{ei-C_n} + n_{ei-T_n} + n_{ei-\sigma}) \quad (1)$$

In(1), $n_{ei-C_n}, n_{ei-T_n} \in [2, \infty], n_{ei-\sigma_n} = 2$, represents the number of components. m refers to the symmetry dimension, whose value is an integer, and $m \in [1, \infty]$.

The scope of application for (1) model is: when the benchmark dimension and components number of the single body symmetry are limited, by (1), we can obtain the symmetry degree; when the three symmetries have an infinite number of benchmark dimensions and symmetry components numbers, we can only make the qualitative judgment of the symmetry degree based on structure symmetrical decomposition diagram, and (1) cannot give the symmetry degree (Bolotov, M.A.et al.,2016); in the mechanical field, (1) can be used to solve the symmetry degree value of any ideal single body structure symmetrical parts. The word "ideal" suggests that the structure symmetry breaking caused by manufacturing errors can be ignored.

1.3 Symmetry degree of mechanical parts and its influence on assembly time

As a kind of simplification of (1), the existing total symmetry angle method can be used to directly calculate the symmetry degree of symmetrical parts with single body structure. However, when there are many symmetrical bodies in the part, this method has the disadvantage that the whole part is regarded as the only symmetrical body.

As an improvement, this paper will distinguish the effect of different symmetry bodies in the parts on the grasp or orientation process, and improve the existing total symmetry angle method from the perspective of multi-body. Set up a unified "symmetry degree - assembly time" prediction model suitable for single symmetry and single body, multi-body and multi-symmetry parts, so as to guide the structural design of parts.

1.3.1 The assembly process affected by parts symmetry

Boothroyd decomposes the mechanical parts manual assembly into: handling, insertion and fixation these three processes, and parts symmetry will affect the handling time. Sturges decomposes part manual assembly into: acquisition process and assembly process, and part symmetry influences the acquisition process. In fact, whether it is "handling" or "acquisition" process, both include the crawl, move, orientation these three operations of the parts (Singh, H.et al., 2016). As the moving process is only related to the size, weight and handling of the parts, it can be considered that the symmetry of the parts mainly affects the process of acquisition and orientation.

1.3.2 The function relationship between the total symmetry angle and assembly time

First of all, define the unified total symmetry angle θ:

$$\theta = \frac{\alpha + \beta}{720^{\circ}} \quad (2)$$

Figure 3 gives the fitting relation curve between the acquisition difficulty coefficient (read by Sturge acquisition difficulty calculator) of 10 parts and its uniformity total symmetry angle θ .

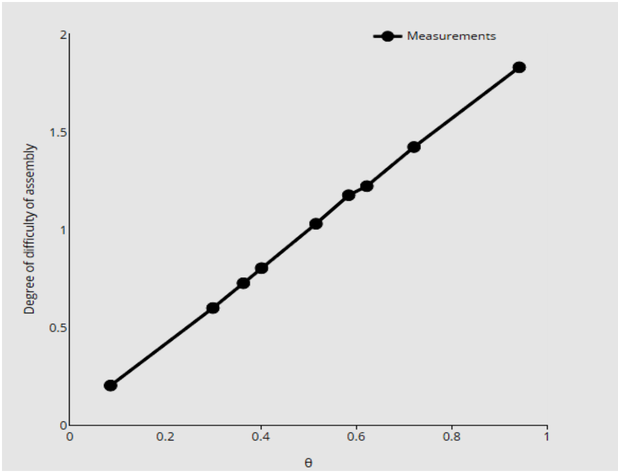

Figure 3. The fitting relation curve between the acquisition difficulty coefficient of 10 parts and (1 Bearing rollers, 2Return shaft and washer, 3Hexagonal prism, 4Rectangular, 5 Cylinder head pin, 6Hexagon head bolt, 7Square head bolt, 8 T font parts, 9Asymmetric parts)

The fitting curve in Figure 3 approximately meets the following line relationship (the fitting curve relevant coefficient R is 0.999, and root mean square error (RMSE) is 0.018).

$$I = 2\theta \quad (3)$$

In (3), I refers to the acquisition difficulty coefficient of mechanical parts. Since that the acquisition difficulty coefficient is linear with parts acquisition time, then the function relationship between the total time T of acquisition and orientation of the parts and the normalized total symmetrical angle is as follows:

$$T = k\theta \quad (4)$$

In (4), $k > 0$ and the value of k is related to the skilled degree of assembler in the machinery manufacturing process. And with the increase of θ, the parts symmetry degree decreases, and the assembly time increases.

1.3.3 Assembly time prediction model of multi-body mechanical parts

This paper first of all puts forward the concept of total correction angle, analyzes the effect of total symmetric angle and total correction angle on assembly time, and deduces the function relation between the total correction angle and total correction angle and the assembly time of parts, and make further modification of (4).

(1) Total correction angle

Similar to the definition of normalized symmetrical angle, set the maximum possible correction angle along the α angle direction to be γ, and the maximum possible correction angle along the β angle direction to be δ, then the total correction angle is:

$$\varphi = \delta + \gamma \quad (5)$$

The total correction angle, in essence, reflects the difference between the contour type φ symmetry body and the directional type symmetry body in symmetry. If the symmetry type of the contour type symmetry body is the same as that of the directional type symmetry body, and the benchmark position coincides and has the consistent symmetry, then the total correction angle is 0. The 6 side surfaces and the 2 end faces of the above hexagonal prism form the contour type symmetry body, and the color surface forms the directional type symmetry body (Mertens, R.et al.,2016). When considering the direction of the mark is upward, the difference between the contour type symmetry body and the directional type symmetry body affects the difficulty of directional operation.

(2) Model building

Set the normalized total symmetrical angle value of the contour type symmetry body to be θ_0, then:

$$\theta_0 = \frac{\alpha_0 + \beta_0}{720^\circ} \quad (6)$$

In the same way, set the normalized total symmetric angle value of the directional type symmetry body to be θ_1, then:

$$\theta_1 = \frac{\alpha_1 + \beta_1}{720^\circ} \quad (7)$$

Set the normalized value of parts total correction angle to be φ, then:

$$\varphi = \frac{\delta + \gamma}{360^\circ} \quad (8)$$

In (8), the angle γ and δ should be calculated combined with a specific example of parts.

The total assembly time of the multi-body symmetrical parts is T, then:

$$T = f(\theta_0, \theta_1, \varphi) \quad (9)$$

Based on the basic premise that "The symmetrical the parts, the higher the assembly efficiency", the concrete expression of the T function of (9) should meet the following four conditions:

(1) For a single symmetric part and a single body multi-symmetric part, there are $\theta_1 = 0$ and $\varphi = 0$. At this point, for an arbitrary θ_0, (9) and (4) must be completely equivalent;

(2) When the structure symmetry of the directional type symmetry body is completely consistent with the contour type symmetry body (The symmetrical form is the same, the symmetrical datum position is coincident, and the symmetry is equal), the total correction angle is 0 (YANO, T.et al., 2017). At this point, $\theta_1 = \theta_0 = 0$ and $\varphi = 0$, and for any θ_0, T must satisfy the following linear increasing relation:

$$T = f(\theta_0, \theta_0, \varphi) = k_1\theta_0 \quad (10)$$

(3) When any of the two independent variables in (9) is fixed, T should have a monotonic relationship with the remaining one variable.

(4) In a completely symmetrical sphere, coat with completely asymmetric patterns. In this particular case, the sphere can be considered as a superposition of the contour type symmetry body and the directional type symmetry body, it also can be directly viewed as asymmetric objects (only considering only the patterns on the sphere). In the two perspectives, the total assembly time of the object should be consistent, namely meeting:

$$T = f(0,1,1) = f(1,0,0) \quad (11)$$

From the Auto2Fit (trial version) software fitting function library, a class of functional families concise in expression and clear in meaning can be found. In the meanwhile, they meet the above four conditions:

$$T = k_1(\theta_0 + \theta_1\varphi^{k_2}) \quad (12)$$

In (12), $k_1 > 0, k_2 > 0$.

Set $k_1 = 1, k_2 = 0.12$, conduct the curve fitting of (11) and (4), and then we can get the general form of assembly time prediction model function of multi-symmetrical parts:

$$T = \theta_0 + \theta_1\varphi^{0.12} \quad (13)$$

3 RESEARCH RESULTS

In order to investigate the effect of the symmetry degree of mechanical parts on the assembly time in the manufacturing process, this paper uses three kinds of parts, including rectangular parts, hexagonal prism parts, and cylinder parts. Two opposite sides and one end of rectangular and hexagonal prismatic parts are painted in different colors, and one end of cylindrical parts is coated with a color (using cross point, single diagonaland double orthogonal diagonal to mark and replace these colors). In the experiment, the height of all parts is 50mm, and the minimum size of the cross section is 25mm.Remove them from the container, and then insert the same 10 square grooves. The groove length is 28mm and the depth is 2mm. The experiment is repeated 10 times, and chronograph is used to time, and the experimental results are shown in table 1.

Table 1. The experiment results

Parts in the experiment	No.	Assembly requirements		Total symmetric angle ($\alpha+\beta$)	Total correction angle ($\gamma+\beta$)	Handling time (s)	Time difference (s)
		Upward side	Front side				
Rectangular	1	Arbitrarily	Arbitrarily	$180^{\circ}+90^{\circ}=270^{\circ}$	$0^{\circ}+0^{\circ}=0^{\circ}$	1.68	0
	2	Arbitrarily	Marked side	$180^{\circ}+180^{\circ}=360^{\circ}$	$0^{\circ}+90^{\circ}=90^{\circ}$	2.14	0.45
	3	Arbitrarily	Single slash side	$180^{\circ}+360^{\circ}=540^{\circ}$	$0^{\circ}+180^{\circ}=180^{\circ}$	2.51	0.82
	4	Cross point side	Arbitrarily	$360^{\circ}+90^{\circ}=450^{\circ}$	$180^{\circ}+0^{\circ}=180^{\circ}$	2.15	0.48
	5	Cross point side	Marked side	$360^{\circ}+180^{\circ}=540^{\circ}$	$180^{\circ}+90^{\circ}=270^{\circ}$	2.57	0.87
	6	Cross point side	Single slash side	$360^{\circ}+360^{\circ}=720^{\circ}$	$180^{\circ}+180^{\circ}=360^{\circ}$	2.74	1.06
Cylinder	7	Arbitrarily	\	$180^{\circ}+0^{\circ}=180^{\circ}$	$0^{\circ}+0^{\circ}=0^{\circ}$	1.52	0
	8	Cross point side	\	$360^{\circ}+0^{\circ}=360^{\circ}$	$180^{\circ}+0^{\circ}=180^{\circ}$	1.99	0.43
Hexagonal prism	9	Arbitrarily	Arbitrarily	$180^{\circ}+60^{\circ}=240^{\circ}$	$0^{\circ}+0^{\circ}=0^{\circ}$	1.78	0
	10	Arbitrarily	Marked side	$180^{\circ}+180^{\circ}=360^{\circ}$	$0^{\circ}+60^{\circ}=60^{\circ}$	2.09	0.36
	11	Arbitrarily	Single slash side	$180^{\circ}+360^{\circ}=540^{\circ}$	$0^{\circ}+180^{\circ}=180^{\circ}$	2.41	0.67
	12	Cross point side	Arbitrarily	$360^{\circ}+60^{\circ}=420^{\circ}$	$180^{\circ}+0^{\circ}=180^{\circ}$	2.29	0.5
	13	Cross point side	Marked side	$360^{\circ}+180^{\circ}=540^{\circ}$	$180^{\circ}+60^{\circ}=240^{\circ}$	2.48	0.72
	14	Cross point side	Single slash side	$360^{\circ}+360^{\circ}=720^{\circ}$	$180^{\circ}+180^{\circ}=360^{\circ}$	2.64	0.87

Notice: α refers to the minimum rotation angle of the part along the vertical direction of the same wheel; β represents the minimum rotation angle of the part along the universal axis; Total symmetric angle is $\alpha+\beta$; Total correction angle is $\gamma+\beta$.

The length, thickness, shaft and hole fit clearance, handling average distance and the insertion depth of the three parts are basically the same. It can be regarded that parts symmetry is the unique reason for the total assembly time difference, and that is to say, parts symmetry degree can influence the assembly and manufacturing of the mechanical parts(Buonamici,F. & Carfagni, M. ,2016).

Given a square plate, and there are different mechanical parts on the plate. Among them, the diameter of the cylinder on the plate is 5mm, and the height is 5mm, symmetrically distributed. Four different multi-symmetrical parts are distributed around it, and in the axial direction of each part, there are through-holes with the diameter of 5mm with a different number but also symmetrically distributed. The length and section size of parts are indicated. The operator needs to complete the assembly operation of aligning the part cylindrical hole on the flat plate and insert it. Table 2 gives the prediction results based on (12), and for comparison, the judgment results of Boothroyd total symmetrical angle method and Sturges method are given.

Table 2 shows that the prediction model can distinguish the assembly efficiency of four kinds of parts, and the conclusion drawn is consistent with the design experience. The assembly time-consuming of cube four-hole parts is less than that of rectangular two-hole parts, which indicates that the symmetrical components can meet the functional and constraint demands, and should use the symmetrical parts as much as possible; the assembly time-consuming of cylindrical two-hole part is less than that of cube two-hole parts, which shows that, compared with the cylinder contour, the symmetry form of cube contour is closer to that of two holes, and therefore it is more likely to interfere with the positioning of two holes. The conclusion suggests that, if the symmetry difference between the directional type symmetry body and the contour type symmetry body is small, it is not conducive for the distinction. Then it is necessary to expand the difference between them, convenient for human or machine recognition.

Table 2. Comparison of assembly time among four kinds of multi-body symmetrical mechanical parts

Type of parts	Comparison of different methods		
	The method in the paper	**Total symmetric angle method**	**Acquisition difficulty coefficient method**
Cylinder two-hole parts	f(0.25,0.25,0.25)=0.672s	1.06s	360°
Cube two-hole parts	f(0.38,0.5,0.25)=0.799s	1.06s	360°
Cube four-hole parts	f(0.38,0.38,0)=0.383s	0.76s	270°
Rectangular two-hole parts	f(0.5,0.5,0)=0.5s	1.05s	360°

4 CONCLUSION

Structural symmetry exists widely in mechanical parts, components and systems. In this paper, based on the theory of space symmetry group, the problems about the symmetry degree of mechanical structure are discussed. In the whole manufacturing process of mechanical parts, the assembly process is essential. This paper focuses on the research on the effect of mechanical structure symmetry on mechanical parts, and puts forward the multi-body mechanical parts assembly time prediction model, and uses the model for the actual case analysis. The results showed that the symmetry degree of parts can affect the assembly and manufacturing of mechanical parts, mainly affecting the assembly time of mechanical parts. Because people can be seen as a machine with a certain degree of dexterity, at the same time, parts of the assembly robot are designed to able to mimic the human hand acquisition, moving and directional and so on operating processes. As a result, for the single symmetry or multi-symmetry parts, (12) model is not only applicable to the evaluation of the efficiency of the manual assembly, but also partially suitable for the assessment of the efficiency of the machine assembly.

5 ACKNOWLEDGMENT

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6 REFERENCES

►Zhou, W. J., Peng, K. Q., & Yu, Y. J. (2016). *Surface roughness measurement and analysis of mechanical parts based on digital holography*. Advances in Manufacturing, 4(3), 217-224.

►Yi, B., Li, X., & Yang, Y. (2016). *Heterogeneous model integration of complex mechanical parts based on semantic feature fusion*. Engineering with Computers, 1-9.

►Yokoki, Y., Yamamori, Y., & Hiraoka, H. (2017). *User Model in the Life Cycle Simulation of Mechanical Parts Based on Prospect Theory*. In Sustainability Through Innovation in Product Life Cycle Design (pp. 353-365). Springer Singapore.

►Bolotov, M. A., Pechenin, V. A., & Murzin, S. P. (2016). *Method for uncertainty evaluation of the spatial mating of high-precision optical and mechanical parts*. Computer Optics, 40(3), 360-369.

►Singh, H., & Jain, P. K. (2016). *Developments in electro-chemical honing (ECH): a review on experimental investigation of precision finishing of* *mechanical parts*. International Journal of Precision Technology, 6(1), 17-41.

►Mertens, R., Boeckmans, B., Thijs, L., Van Humbeeck, J., & Kruth, J. P. (2016, January). *Effect of Hot Isostatic Pressing on mechanical properties and dimensional accuracy of intentionally porous Ti6Al4V parts made by Selective Laser Melting*. In Contributed papers from Materals Science and Technology 2016 (MS&T16) (pp. 35-42).

► YANO, T., MATSUNAGA, T., ARIOKA, M., KOYAMA, K., & TAKEUCHI, T. (2017). *Development of Mechanical CBM System for Electromagnetically Actuated Vacuum Circuit Breaker*. Electronics and Communications in Japan, 100(2), 50-57.

►Buonamici, F., & Carfagni, M. (2016, August). *Reverse Engineering of Mechanical Parts: A Brief Overview of Existing Approaches and Possible New Strategies*. In ASME 2016 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference (pp. V01BT02A003-V01BT02A003). American Society of Mechanical Engineers.