CLOSED-LOOP SUPPLY CHAIN COORDINATE: QUALITY AND RECOVERY

Jian Tan[1]

ABSTRACT: By using game theory, we have studied product quality and waste product recovery rate decision problems of CLSC which contains a manufacture and a retailer, the centralized decision model, retailer recovery and manufacturer recovery model are analyzed. Research results show that the quality level, recovery rate, demand and profits all are the highest in the centralized decision model while they are higher in the manufacture recovery mode of decentralized decision model. At the same time, the simulations verify the relevant conclusions.

KEY WORDS: closed-loop supply chain, coordinate, quality level, recovery

1 INTRODUCTION

With the natural resources gradually exhausted, environment problem is becoming increasingly prominent, the closed-loop supply chain attract extensive attention of all circles of the society, many countries have also increased legislative efforts, formulate corresponding laws and regulations which require companies to make full use of waste products. Closed-loop supply chain can not only solve the problem of resource shortage and environmental pollution, but also can bring more economic benefits and social benefits for the enterprise. But with frequent outbreak of product quality, product quality problem have seriously affected the efficiency of supply chain, and quality competition of enterprises is gradually transformed for the quality competition between supply chains. Therefore, as the core enterprise of closed-loop supply chain, through the selection of effective strategies, coordinating the relationship between the core enterprise, optimizing the closed-loop supply chain, improving the recovery efficiency and quality level, to achieve a win-win is of important significance.

From the perspective of supply chain, the problem of quality management has caused the attention of some scholars. Chao, Iravani et al. (2009) studied impact of products recall cost sharing on product quality improvement in a supply chain consisting of a supplier and a retailer [1]. Brekke, Siciliani et al. (2010) studied the relationship between competition and quality within a spatial competition framework where firms compete in prices and quality [2].

A non-cooperative dynamic game is formulated in which a single supplier collaborates with two manufacturers on design quality improvements for their respective products by El Ouardighi and Kim (2010), the manufacturers compete for market demand both on price and design quality [3]. Xie, Wang et al. (2011) consider quality improvement in a given segment of the market, shared by two supplier–manufacturer supply chains which offer a given product at the same price but compete on quality, the mechanism on the selection of supply chain structures and quality improvement strategies of the two supply chains is described [4]. Xie, Yue et al. (2011) investigated quality investment and price decision of a make-to-order (MTO) supply chain with uncertain demand in international trade. In contrast to the general assumption that players in a supply chain are risk neutral in quality investment and price decision, they consider the risk-averse behaviour of the players in three different supply chain strategies [5]. Lambertini and Tampieri (2012) model a vertically differentiated duopoly with quantity-setting firms as an extended game in which firms no cooperatively choose the timing of moves at the quality stage, to show that at the subgame, perfect equilibrium sequential play obtains, with the low-quality firm taking the leader’s role[6]. Because the literatures on channel coordination has not considered quality uncertainty in designing a contract of alliance, Lee, Rhee et al.(2013) bridged the gap between these two streams of literature by explicitly considering quality uncertainty in a coordination framework[7]. Under the assumption that the quality of the products and sales effort effects on demand, Ma, Wang et al. (2013) used two stages game to study the problem of supply chain coordination [8-9]. El Ouardighi (2014) established the wholesale price contract and revenue sharing contract models under dynamic conditions of supply chain quality management by

1. School of Management Science, Guizhou University of Finance and Economics, Guiyang 550025, China

two stages game [10]. Yoo (2014) discussed the quality and the return of the goods decision of supplier in supply chain by using the reference quality [11].

As can be seen from the above literatures, important researches have been accomplished for the quality management of supply chain, but there are few research on the quality management for the closed-loop supply chain. Zhongkai Xiong (2007) just discussed recycling products quality problems while ignored new products quality [12]. Jiaping Xie(2012) only considered the impact of quality on the manufacturer in the case of recycling, without regard to the impact of the quality of the entire supply chain[13]. On the basis of the literatures, we will study closed-loop supply chain price and quality problems of coordination, to help enterprises of closed-loop supply chain optimization decision, provide meaningful theoretical guidance for the practice of the closed-loop supply chain.

2 MODEL DESCRIPTION

A closed-loop supply chain (CLSC) contains a manufacture and a retailer in our models. The manufacture sells the products which quality is x with wholesale price w to the retailer, the retailer sells the products to the consumer with retail price p. Waste products recycled for remanufacturing, the recovery rate is ζ, the corresponding cost is ηζ^2, where η denotes the cost parameter. Suppose the waste products recycled are used for current remanufacture, and the quality of the products is in line with the new products. Suppose ϱ=c-c_r, which c means the unit cost of new product, and c_r is unit cost of remanufacture product, then ϱ means the cost savings for the unit remanufacture product.

Following Dixit et al.(1999)[14], we assume that the demand information is symmetrically known to both CLSC members, and the demand function is linear in price and quality:

$$d(p,x) = \phi - \beta p + \lambda x \qquad (1)$$

Where ϕ is the base market size, x means quality level, β(λ) denotes the demand responsiveness to CLSC's own price (quality).

According to Banker, Khosla et al. (1998) [15], the cost function for CLSC is given by

c(d,x)=(c+ϵx)d+δx^2 (2)

Thus, the quality level selected by a firm affects total costs in two ways. First, investment in a quality improvement program increases fixed production costs δx^2, which is increasing and convex in the quality level x, and δ is the fixed cost parameter. Second, the quality level also has an impact on the production cost per unit. Specifically, c denotes the variable production cost per unit not including the quality related costs. Given a quality level x selected by CLSC, the unit variable cost increases by ϵx, where ϵ>0. We assume λ/β>ϵ, i.e., λ>βϵ.

Let〚 π〛^R, π^Mand π^T denote the profit of the retailer, the manufacturer and the CLSC, respectively. We use subscripts T, R and M to denote centralized decision, retailer recycling and manufacturer recycling model, respectively. Superscript * denotes the optimal value.

3 PRICE AND QUALITY EQUILIBRIUM UNDER DIFFERENT CLSC STRUCTURES

3.1 Centralized decision model

The manufacturer and the retailer are considered as a whole in this model. They choose optimal retail price, quality level and recovery rate to maximum the overall profit of the CLSC. From Eq.(1) and Eq.(2), the profit function for the CLSC as follow:

$$\pi_T(p,x,\zeta) = (p + \varrho\zeta - (c + \epsilon x))(\phi - \beta p + \lambda x) - \eta\zeta^2 - \delta x^2 \qquad (3)$$

he first-order conditions characterizing equilibrium retail price, quality level and recovery rate are:

$$\frac{\partial \pi_T}{\partial p} = c\beta - 2p\beta + x\beta\epsilon + x\lambda - \beta\zeta\varrho + \phi = 0 \qquad (4)$$

$$\frac{\partial \pi_T}{\partial x} = -c\lambda + p(\beta\epsilon + \lambda) - 2x(\delta + \epsilon\lambda) + \zeta\lambda\varrho - \epsilon\phi = 0 \qquad (5)$$

$$\frac{\partial \pi_T}{\partial \zeta} = -2\zeta\eta + \varrho(-p\beta + x\lambda + \phi) = 0 \qquad (6)$$

Since $\frac{\partial^2 \pi_T}{\partial p^2} = -2\beta < 0$, $\frac{\partial^2 \pi_T}{\partial x^2} = -2\delta - 2\epsilon\lambda < 0$, $\frac{\partial^2 \pi_T}{\partial \zeta^2} = -2\eta < 0$, the profit function given x and ζ (x and p ,p and ζ) is strictly concave in $p(\zeta, x)$. Note that the Hessian of π_T is negative definite for all values of p,ζ and x if $-4\beta\delta\eta + \beta^2\delta\varrho^2 + \eta(\lambda - \beta\epsilon)^2 < 0$(this condition also needs to be satisfied in the following).

Solving Eq.(4)~ Eq.(6), we obtain the equilibrium retail price, quality level and recovery rate:

$$p_T^* = -\frac{c\eta(\lambda^2 - \beta(2\delta + \epsilon\lambda)) + (\epsilon\eta(\beta\epsilon - \lambda) + \delta(-2\eta + \beta\varrho^2))\phi}{A} \qquad (7)$$

$$x_T^* = \frac{\eta(\lambda - \beta\epsilon)B}{A} \qquad (8)$$

$$\zeta_T^* = \frac{\beta\delta\varrho B}{A} \qquad (9)$$

Where $A = -\eta(\lambda - \beta\epsilon)^2 + 2\beta\delta\eta - \beta^2\delta\varrho^2$, $B = \phi - c\beta$.

After substituting Eq.(7)~Eq.(9) into Eq.(3), we have

$$d_T^* = \frac{2\beta\delta\eta B}{A} \quad (10)$$

$$\pi_T^* = \frac{\delta\eta B^2}{A} \quad (11)$$

3.2 Manufacture recovery Stackelberg model

The competition between the manufacture and retailer takes place in the following sequence in time:

(i)The manufacture recoveries the waste products for remanufacturing and apply new raw material for manufacturing, sells the new products to the retailer. He selects quality level, wholesale price and recovery rate to maximum his profits.

(ii) The retailer sells the products to the consumer. He selects retailer price to maximum his profits.

We express the profits of the CLSC members as follows:

$$\pi_M^M(w,x,\zeta) = (w + \varrho\zeta - (c + \epsilon x))(\phi - \beta p + \lambda x) - \eta\zeta^2 - \delta x^2/2 \quad (12)$$

$$\pi_M^R(p) = (p - w)(\phi - \beta p + \lambda x) \quad (13)$$

The manufacturer takes the retailer's reaction into consideration when choosing its strategy. The retailer's reaction function for a given w, x and ζ can be derived from the first-order derivative of π_M^R in Eq.(13):

$$\frac{\partial \pi_M^R}{\partial p} = -2p\beta + w\beta + x\lambda + \phi = 0 \quad (14)$$

Solving Eq.(14),we obtain the equilibrium price:

$$p_M^* = \frac{w\beta + x\lambda + \phi}{2\beta} \quad (15)$$

Substituting Eq.(15) into Eq.(12), the first-order conditions characterizing equilibrium w, x and ζ are:

$$\frac{\partial \pi_M^M}{\partial x} = \frac{1}{2}(-c\lambda + w(\beta\epsilon + \lambda) - 2x(\delta + \epsilon\lambda) + \zeta\lambda\varrho - \epsilon\phi) = 0 \quad (16)$$

$$\frac{\partial \pi_M^M}{\partial \zeta} = \frac{1}{2}\left(-4\zeta\eta + \varrho(-w\beta + x\lambda + \phi)\right) = 0 \quad (17)$$

$$\frac{\partial \pi_M^M}{\partial w} = \frac{1}{2}(c\beta - 2w\beta + x\beta\epsilon + x\lambda - \beta\zeta\varrho + \phi) = 0 \quad (18)$$

Solving Eq.(16)~Eq.(18), we find that the manufacturing's optimal wholesale price, recovery rate and quality level are:

$$w_M^* = \frac{c\beta^2\delta\varrho^2 - 2cA + (2\epsilon\eta(\beta\epsilon - \lambda) - \delta(4\eta - \beta\varrho^2))B}{2A + H} \quad (19)$$

$$\zeta_M^* = \frac{\beta\delta\varrho B}{2A + H} \quad (20)$$

$$x_M^* = \frac{2\eta(\lambda - \beta\epsilon)B}{2A + H} \quad (21)$$

Where $H = \beta^2\delta\varrho^2$.

Substituting Eq.(19) ~ Eq.(21) into Eq.(15), we obtain the retail price:

$$p_M^* = \frac{cH - 2cA - (2\epsilon\eta(\beta\epsilon - \lambda) - \delta(6\eta - \beta\varrho^2))B}{2A + H} \quad (22)$$

Substituting Eq.(19) ~ Eq.(22) into Eq.(12) and Eq.(13), we can get:

$$\pi_M^{M*} = \frac{\delta\eta B^2}{2A + H} \quad (23)$$

$$\pi_M^{R*} = \frac{4\beta^2\delta^2\eta B^2}{(2A + H)^2} \quad (24)$$

$$d_M^* = \frac{2\beta\delta\eta B}{2A + H} \quad (25)$$

$$\pi_M^{T*} = \frac{\delta\eta(\beta\delta(4\eta + \beta\varrho^2) - 2A)B^2}{(2A + H)^2} \quad (26)$$

3.3 Retailer recovery Stackelberg model

The competition between the manufacture and retailer takes place in the following sequence in time:

(i) The manufacture uses the waste products recovered by retailer and new raw material for manufacturing, and sell the new products to the retailer. He selects quality level and wholesale price to maximum his profits.

(ii) The retailer recoveries waste products and resell them with price b per unit to the manufacture, sells the products to the consumer with price b. He selects retailer price and recovery rate to maximum his profits.

We express the profits of the CLSC members as follows:

$$\pi_R^M(w,x) = (w + (\varrho - b)\zeta - (c + \epsilon x))(\phi - \beta p + \lambda x) - \delta x^2/2 \quad (27)$$

$$\pi_R^R(p,\zeta) = (p - w + b\zeta)(\phi - \beta p + \lambda x) - \eta\zeta^2 \quad (28)$$

Similar to the 3.2 section, take the first derivative of Eq.(28), we have:

$$\frac{\partial \pi_R^R}{\partial p} = -2p\beta + w\beta - b\beta\zeta + x\lambda + \phi = 0 \quad (29)$$

$$\frac{\partial \pi_R^R}{\partial \zeta} = -2\zeta\eta + b(-p\beta + x\lambda + \phi) = 0 \quad (30)$$

Note that the Hessian matrix of π_R^R is negative definite for all values of p and ζ if $b^2\beta - 4\eta < 0$.From

Eq.(29) and Eq.(30),we find that

$$p = \frac{-2w\beta\eta + (b^2\beta - 2\eta)(x\lambda + \phi)}{\beta(b^2\beta - 4\eta)} \quad (31)$$

$$\zeta = \frac{b(w\beta - x\lambda - \phi)}{b^2\beta - 4\eta} \quad (32)$$

Substituting Eq.(31) and Eq.(32) into Eq.(27), we find that the manufacturing's optimal wholesale price and quality level are:

$$w_R^* = \frac{b^2\beta\delta(c\beta-\phi)+2b\beta\delta\varrho\phi-2\eta(c(2\beta\delta+\beta\epsilon\lambda-\lambda^2)+(2\delta+\epsilon(-\beta\epsilon+\lambda))\phi)}{2(\eta\lambda^2-2\beta\eta(2\delta+\epsilon\lambda)+\beta^2(\epsilon^2\eta+b\delta\varrho))} \quad (33)$$

$$x_R^* = \frac{\eta(\beta\epsilon-\lambda)(c\beta-\phi)}{\eta\lambda^2-2\beta\eta(2\delta+\epsilon\lambda)+\beta^2(\epsilon^2\eta+b\delta\varrho)} \quad (34)$$

Substituting Eq.(33) and Eq.(34) into Eq.(27) ,

Eq.(28) ,Eq.(31) and Eq.(32) ,then we can have:

$$p_R^* = \frac{c\eta(\lambda^2-\beta(\delta+\epsilon\lambda))+(\epsilon\eta(\beta\epsilon-\lambda)+\delta(-3\eta+b\beta\varrho))\phi}{\eta\lambda^2-2\beta\eta(2\delta+\epsilon\lambda)+\beta^2(\epsilon^2\eta+b\delta\varrho)} \quad (35)$$

$$d_R^* = \frac{\beta\delta\eta(c\beta-\phi)}{\eta\lambda^2-2\beta\eta(2\delta+\epsilon\lambda)+\beta^2(\epsilon^2\eta+b\delta\varrho)} \quad (36)$$

$$\zeta_R^* = \frac{b\beta\delta(c\beta-\phi)}{2(\eta\lambda^2-2\beta\eta(2\delta+\epsilon\lambda)+\beta^2(\epsilon^2\eta+b\delta\varrho))} \quad (37)$$

$$\pi_R^{M*} = -\frac{\delta\eta(-c\beta+\phi)^2}{2(\eta\lambda^2-2\beta\eta(2\delta+\epsilon\lambda)+\beta^2(\epsilon^2\eta+b\delta\varrho))} \quad (38)$$

$$\pi_R^{R*} = \frac{\beta\delta^2(4\eta-b^2\beta)\eta(-c\beta+\phi)^2}{4(\eta\lambda^2-2\beta\eta(2\delta+\epsilon\lambda)+\beta^2(\epsilon^2\eta+b\delta\varrho))^2} \quad (39)$$

$$\pi_R^{T*} = -\frac{\delta\eta(b^2\beta^2\delta+2\eta(\beta^2\epsilon^2+\lambda^2-2\beta(3\delta+\epsilon\lambda))+2b\beta^2\delta\varrho)(-c\beta+\phi)^2}{4(\eta\lambda^2-2\beta\eta(2\delta+\epsilon\lambda)+\beta^2(\epsilon^2\eta+b\delta\varrho))^2} \quad (40)$$

From Eq. (38) ~Eq. (40), we can figure out that the profits are maximum while $b = 0$. Then Eq. (35) ~Eq. (40) rewrite as follows:

$$w_R^* = -\frac{c(\lambda^2-\beta(2\delta+\epsilon\lambda))+(-2\delta+\epsilon(\beta\epsilon-\lambda))\phi}{\frac{A+H}{\eta}} \quad (41)$$

$$x_R^* = \frac{(\lambda-\beta\epsilon)B}{\frac{A+H}{\eta}} \quad (42)$$

$$p_R^* = -\frac{c(\lambda^2-\beta(\delta+\epsilon\lambda))+(-3\delta+\epsilon(\beta\epsilon-\lambda))\phi}{\frac{A+H}{\eta}} \quad (43)$$

$$\pi_R^{M*} = \frac{\delta B^2}{\frac{A+H}{\eta}} \quad (44)$$

$$\pi_R^{R*} = \frac{\beta\delta^2 B^2}{(\frac{A+H}{\eta})^2} \quad (45)$$

$$d_R^* = \frac{\beta\delta B}{\frac{A+H}{\eta}} \quad (46)$$

$$\pi_R^{T*} = \frac{\delta(-\beta^2\epsilon^2-\lambda^2+2\beta(3\delta+\epsilon\lambda))B^2}{2(\frac{A+H}{\eta})^2} \quad (47)$$

$$\zeta_R^* = 0 \quad (48)$$

4 COMPARATIVE ANALYSIS

Proposition1: (i) $x_T^* > x_M^* > x_R^*$; (ii) $\frac{dx_T^*}{d\lambda} > 0, \frac{dx_M^*}{d\lambda} > 0, \frac{dx_R^*}{d\lambda} > 0$; $\frac{dx_T^*}{d\delta} < 0, \frac{dx_M^*}{d\delta} < 0, \frac{dx_R^*}{d\delta} < 0$; $\frac{dx_T^*}{d\eta} < 0, \frac{dx_M^*}{d\eta} < 0, \frac{dx_R^*}{d\eta}=0$.

Proof:

$$x_T^* - x_M^* = \frac{\beta^2\delta\eta(\lambda-\beta\epsilon)\varrho^2 B}{A(2A+H)} > 0, x_M^* - x_R^* = \frac{\eta\beta^2\delta(\lambda-\beta\epsilon)\varrho^2 B}{(2A+H)(A+H)} > 0.$$

$$\frac{dx_T^*}{d\lambda} = \frac{\eta(\eta(\lambda-\beta\epsilon)^2+\beta\delta(4\eta-\beta\varrho^2))B}{A^2} > 0,$$

$$\frac{dx_M^*}{d\lambda} = \frac{2\eta(2\eta(\lambda-\beta\epsilon)^2+\beta\delta(8\eta-\beta\varrho^2))B}{(2A+H)^2} > 0,$$

$$\frac{dx_R^*}{d\lambda} = \frac{(4\beta\delta+(\lambda-\beta\epsilon)^2)B}{(\frac{A+H}{\eta})^2} > 0;$$

$$\frac{dx_T^*}{d\delta} = \frac{\beta\eta(\beta\epsilon-\lambda)(4\eta-\beta\varrho^2)B}{A^2} < 0, \frac{dx_M^*}{d\delta} = \frac{2\beta\eta(\beta\epsilon-\lambda)(8\eta-\beta\varrho^2)B}{(2A+H)^2} < 0, \frac{dx_R^*}{d\delta} = \frac{4\beta(\beta\epsilon-\lambda)B}{(\frac{A+H}{\eta})^2} < 0$$

$$;\frac{dx_T^*}{d\eta} = \frac{\beta^2\delta(\beta\epsilon-\lambda)\varrho^2 B}{A^2} < 0, \frac{dx_M^*}{d\eta} = \frac{2\beta^2\delta(\beta\epsilon-\lambda)\varrho^2 B}{(2A+H)^2} < 0, \frac{dx_R^*}{d\eta}=0.$$

The above proposition indicates that the quality level of centralized mode is the highest while the quality level of the retailer recovery mode is the lowest. The quality level increases with the increase of the coefficient of demand responsiveness to CLSC's own quality, and decreases with the increase of fixed quality cost parameter. In the centralized decision and the manufacturer recycling mode, the quality level decreases with the increase of the recovery cost parameter, and the quality level of the retailer recovery mode has nothing to do with this parameter. The simulation verifies the above conclusion, as shown in Fig. 1. From Fig.1, we can see that, with the increase of λ, the quality gap between the models is increasing, but when η is increasing, the result is the opposite.

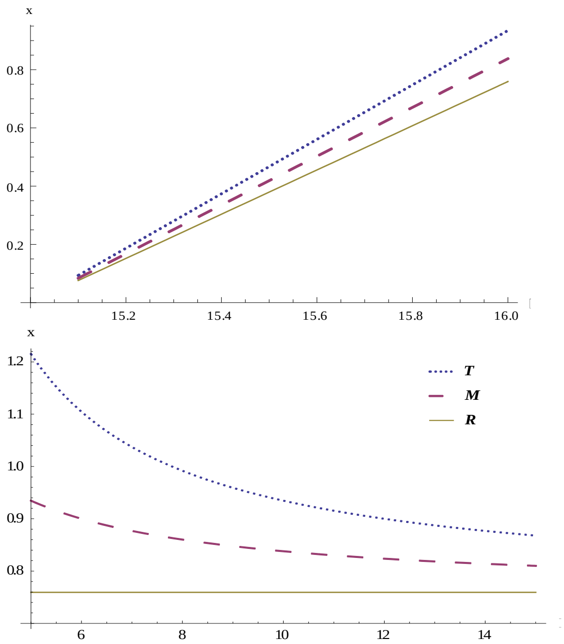


Figure.1. Product quality level vary with parameter
λ and η ($\phi = 10000, \beta = 30, c = 60, \epsilon = 0.5, \lambda = 16, \delta = 90, \eta = 10, \varrho = 0.3$)

.

Proposition 1 tells us that in order to improve the product quality of the supply chain, increasing λ, reducing η and δ are important way. For decentralized supply chain, the product quality is the highest in manufacture recovery mode.

Proposition 2 : (i)$\zeta_T^* > \zeta_M^* > \zeta_R^*$;(ii) $\frac{d\zeta_T^*}{d\lambda} > 0$, $\frac{d\zeta_M^*}{d\lambda} > 0$, $\frac{d\zeta_R^*}{d\lambda} = 0$;
$\frac{d\zeta_T^*}{d\delta} < 0, \frac{d\zeta_M^*}{d\delta} < 0, \frac{d\zeta_R^*}{d\delta} = 0; \frac{d\zeta_T^*}{d\eta} < 0, \frac{d\zeta_M^*}{d\eta} < 0, \frac{d\zeta_R^*}{d\eta} = 0.$

Proof: $\zeta_T^* - \zeta_M^* = \frac{\beta\delta\eta(-(\lambda-\beta\epsilon)^2+4\beta\delta))\varrho B}{A(2A+H)} > 0$;
$\frac{d\zeta_T^*}{d\lambda} = \frac{2\beta\delta\eta(\lambda-\beta\epsilon)\varrho B}{A^2} > 0, \quad \frac{d\zeta_M^*}{d\lambda} = \frac{4\beta\delta\eta(\lambda-\beta\epsilon)\varrho B}{(2A+H)^2} > 0, \quad \frac{d\zeta_R^*}{d\lambda} = 0$;
$\frac{d\zeta_T^*}{d\delta} = \frac{\beta\eta(-\beta\epsilon+\lambda)^2\varrho B}{A^2} < 0, \frac{d\zeta_M^*}{d\delta} = -\frac{2\beta\eta(-\beta\epsilon+\lambda)^2\varrho B}{(2A+H)^2} < 0, \frac{d\zeta_R^*}{d\delta} = 0$;
$\frac{d\zeta_T^*}{d\eta} = -\frac{\beta\delta(-(\lambda-\beta\epsilon)^2+4\beta\delta)\varrho B}{A^2} < 0$,
$\frac{d\zeta_M^*}{d\eta} = -\frac{2\beta\delta(-(\lambda-\beta\epsilon)^2+4\beta\delta)\varrho B}{(2A+H)^2} < 0, \frac{d\zeta_R^*}{d\eta} = 0.$

Proposition 2 indicates that the recovery rate of centralized mode is the highest while the recovery rate of the retailer recovery mode is the lowest. In centralized decision model and manufacturer recovery mode, the recovery rate increases with the increase of the coefficient of demand responsiveness to CLSC's own quality, and decreases with the increase of fixed quality cost parameter and the recovery cost parameter while the recovery rate of retailer recovery mode has nothing to do with the parameters. As shown in Fig.2, note that ζ_R^ curve is a straight line and coincides with the coordinate axis.

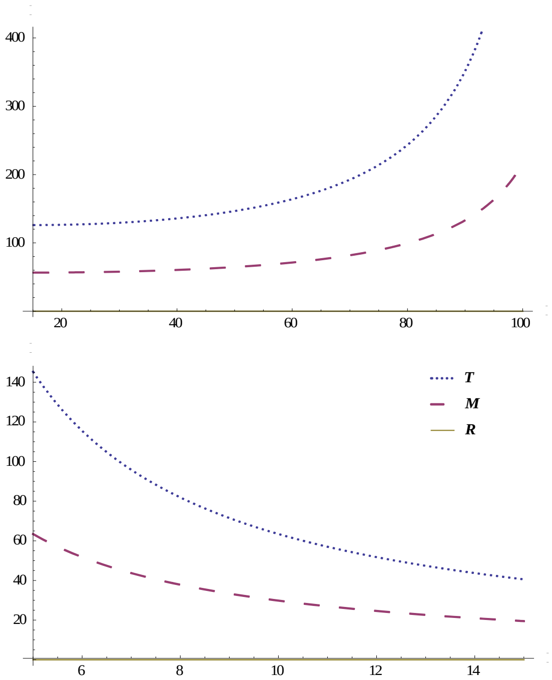


Figure.2. Recovery rate vary with parameter λ and $\eta(\phi = 10000, \beta = 30, c = 60, \epsilon = 0.5, \lambda = 16, \delta = 90, \eta = 10, \varrho = 0.3)$

Proposition 2 tells us that in order to improve the product recovery rate of the supply chain, increasing λ, reducing η and δ also are important way. For decentralized supply chain, the manufacture recovery mode is advantageous to improve product recovery rate.

Proposition 3 : (i)$d_T^* > d_M^* > d_R^*$; (ii) $\frac{dd_T^*}{d\lambda} > 0$, $\frac{dd_M^*}{d\lambda} > 0$, $\frac{dd_R^*}{d\lambda} > 0$;
$\frac{dd_T^*}{d\delta} > 0, \frac{dd_M^*}{d\delta} > 0, \frac{dd_R^*}{d\delta} > 0; \frac{dd_T^*}{d\eta} < 0, \frac{dd_M^*}{d\eta} < 0, \frac{dd_R^*}{d\eta} = 0.$

Proof: $d_T^* - d_M^* = \frac{2\beta\delta\eta^2(-(\lambda-\beta\epsilon)^2+4\beta\delta))B}{A(2A+H)} > 0$,
$d_T^* - d_R^* = \frac{\beta\delta(-(\lambda-\beta\epsilon)^2+\beta\delta(4\eta+\beta\varrho^2))B}{(\frac{A+H}{\eta})A} > 0$,
$d_M^* - d_R^* = \frac{\beta^2\delta^2\varrho^2 B}{(\frac{A+H}{\eta})(2A+H)} > 0. \frac{dd_T^*}{d\lambda} = \frac{4\beta\delta\eta^2(\lambda-\beta\epsilon)B}{A^2} > 0$,
$\frac{dd_M^*}{d\lambda} = \frac{8\beta\delta\eta^2(\lambda-\beta\epsilon)B}{(2A+H)^2} > 0, \quad \frac{dd_R^*}{d\lambda} = \frac{2\beta\delta(\lambda-\beta\epsilon)B}{(\frac{A+H}{\eta})^2} > 0$;
$\frac{dd_T^*}{d\delta} = \frac{2\beta\eta^2(-\beta\epsilon+\lambda)^2B}{A^2} > 0, \frac{dd_M^*}{d\delta} = \frac{4\beta\eta^2(-\beta\epsilon+\lambda)^2B}{(2A+H)^2} > 0, \frac{dd_R^*}{d\delta} = \frac{\beta(-\beta\epsilon+\lambda)^2B}{(\frac{A+H}{\eta})^2} > 0$
; $\frac{dd_T^*}{d\eta} = -\frac{2\beta^2\delta^2\varrho^2 B}{A} < 0, \frac{dd_M^*}{d\eta} = -\frac{2\beta^2\delta^2\varrho^2 B}{(2A+H)^2} < 0, \frac{dd_R^*}{d\eta} = 0.$

Proposition 3 indicates that the demand of centralized mode is the highest while the demand of the retailer recovery mode is the lowest. The demand increases with the increase of the coefficient of demand responsiveness to CLSC's own quality, and decreases with the increase of fixed quality cost parameter. In the centralized decision and the manufacturer recycling mode, the demand decreases with the increase of the recovery cost parameter, and the demand of the retailer recovery mode has nothing to do with this parameter. As shown in fig.3, for decentralized supply chain, the manufacture recovery mode is advantageous to increase product demanding.

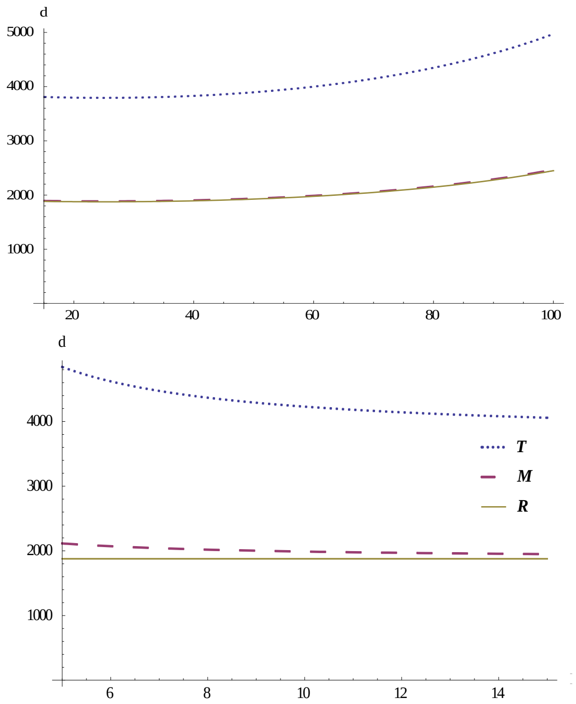


Figure.3. Demand vary with parameter λ and $\eta(\phi = 10000, \beta = 30, c = 60, \epsilon = 0.5, \lambda = 16, \delta = 90, \eta = 10, \varrho = 0.3)$

.

Proposition 4 :(i) $\pi_T^{T*} > \pi_M^{T*} > \pi_R^{T*}$; $\pi_M^{M*} > \pi_R^{M*}$; $\pi_M^{R*} > \pi_R^{R*}$

(ii) $\frac{d\pi_T^{T*}}{d\lambda} > 0$, $\frac{d\pi_M^{T*}}{d\lambda} > 0$, $\frac{d\pi_R^{T*}}{d\lambda} > 0$; $\frac{d\pi_T^{T*}}{d\delta} < 0$, $\frac{d\pi_M^{T*}}{d\delta} < 0$, $\frac{d\pi_R^{T*}}{d\delta} < 0$; $\frac{d\pi_T^{T*}}{d\eta} < 0$, $\frac{d\pi_M^{T*}}{d\eta} < 0$, $\frac{d\pi_R^{T*}}{d\eta} = 0$;

$\frac{d\pi_M^{M*}}{d\lambda} > 0, \frac{d\pi_R^{M*}}{d\lambda} > 0, \frac{d\pi_M^{M*}}{d\delta} < 0, \frac{d\pi_R^{M*}}{d\delta} < 0, \frac{d\pi_M^{M*}}{d\eta} < 0, \frac{d\pi_R^{M*}}{d\eta} = 0.$

$\frac{d\pi_M^{R*}}{d\lambda} > 0, \frac{d\pi_R^{R*}}{d\lambda} > 0, \frac{d\pi_M^{R*}}{d\delta} < 0, \frac{d\pi_R^{R*}}{d\delta} < 0, \frac{d\pi_M^{R*}}{d\eta} < 0, \frac{d\pi_R^{R*}}{d\eta} = 0.$

Proof: $\pi_M^{M*} - \pi_R^{M*} = \frac{\beta^2\delta^2\varrho^2B^2}{2\left(\frac{A+H}{\eta}\right)(2A+H)} > 0$;

$$\pi_M^{R*} - \pi_R^{R*} = \frac{\beta^2\delta^2\varrho^2\left(-4\eta(\lambda-\beta\epsilon)^2+\beta\delta(16\eta-\beta\varrho^2)\right)B^2}{\left(\frac{A+H}{\eta}\right)^2(2A+H)^2} > 0;$$

$$\pi_T^{T*} - \pi_M^{T*} = \frac{B^2\delta\eta\left(6A^2+H^2+A\beta\delta(-4\eta+3\beta\varrho^2)\right)}{A(2A+H)^2} > 0,$$

$$\pi_M^{T*} - \pi_R^{T*} = \frac{B^2\eta(-2(A+H)^2\delta(2A-\beta\delta(4\eta+\beta\varrho^2))-(2A+H)^2\eta\delta[\beta^2\epsilon^2+\lambda^2-2\beta[3\delta+\epsilon\lambda]])}{2(A+H)^2(2A+H)^2} > 0$$

;

$$\frac{d\pi_T^{T*}}{d\lambda} = \frac{2\delta\eta^2(\lambda-\beta\epsilon)(-c\beta+\phi)^2}{A^2} > 0,$$

$$\frac{d\pi_M^{T*}}{d\lambda} = \frac{4\delta\eta^2(\lambda-\beta\epsilon)(-2\eta(\lambda-\beta\epsilon)^2+\beta\delta(16\eta-\beta\varrho^2))B^2}{(2A+H)^2} > 0,$$

$$\frac{d\pi_R^{T*}}{d\lambda} = \frac{\delta(\lambda-\beta\epsilon)(-(\lambda-\beta\epsilon)^2+8\beta\delta)B^2}{(\frac{A+H}{\eta})^2} > 0;$$

$$\frac{d\pi_T^{T*}}{d\delta} = -\frac{\eta^2(\lambda-\beta\epsilon)^2B^2}{A^2} < 0,$$

$$\frac{d\pi_M^{T*}}{d\delta} = -\frac{2\eta^2(\lambda-\beta\epsilon)^2\left(-2\eta(\lambda-\beta\epsilon)^2+\beta\delta(16\eta-\beta\varrho^2)\right)B^2}{(2A+H)^2} < 0, \frac{d\pi_R^{T*}}{d\delta} = -\frac{(\lambda-\beta\epsilon)^2(-(\lambda-\beta\epsilon)^2+8\beta\delta)B^2}{2\left(\frac{A+H}{\eta}\right)^2} < 0$$

;

$$\frac{d\pi_T^{T*}}{d\eta} = -\frac{\beta^2\delta^2\varrho^2B^2}{A^2} < 0,$$

$$\frac{d\pi_M^{T*}}{d\eta} = \frac{\beta^2\delta^2\varrho^2(-2\eta(\lambda-\beta\epsilon)^2+\beta\delta(16\eta-\beta\varrho^2))B^2}{(2A+H)^2} < 0, \frac{d\pi_R^{T*}}{d\eta} = 0.$$

$$\frac{d\pi_M^{M*}}{d\lambda} = \frac{4\delta\eta^2(-\beta\epsilon+\lambda)(-c\beta+\phi)^2}{(2\eta\lambda^2-4\beta\eta(2\delta+\epsilon\lambda)+\beta^2(2\epsilon^2\eta+\delta\varrho^2))^2} > 0,$$

$$\frac{d\pi_R^{M*}}{d\lambda} = -\frac{\delta(\beta\epsilon-\lambda)(-c\beta+\phi)^2}{\left(\beta^2\epsilon^2+\lambda^2-2\beta(2\delta+\epsilon\lambda)\right)^2} > 0,$$

$$\frac{d\pi_M^{M*}}{d\delta} = -\frac{2\eta^2(-\beta\epsilon+\lambda)^2(-c\beta+\phi)^2}{\left(2\eta\lambda^2-4\beta\eta(2\delta+\epsilon\lambda)+\beta^2(2\epsilon^2\eta+\delta\varrho^2)\right)^2} < 0,$$

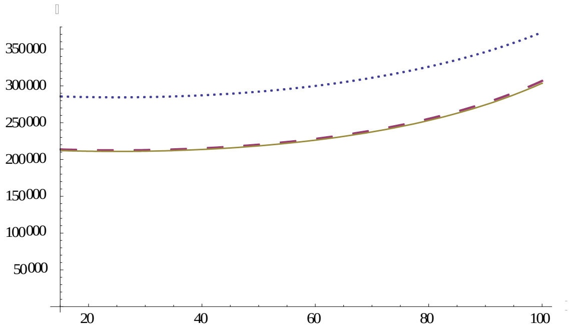
$$\frac{d\pi_R^{M*}}{d\delta} = -\frac{(-\beta\epsilon+\lambda)^2(-c\beta+\phi)^2}{2\left(\beta^2\epsilon^2+\lambda^2-2\beta(2\delta+\epsilon\lambda)\right)^2} < 0,$$

$$\frac{d\pi_M^{M*}}{d\eta} = -\frac{\beta^2\delta^2\varrho^2(-c\beta+\phi)^2}{\left(2\eta\lambda^2-4\beta\eta(2\delta+\epsilon\lambda)+\beta^2(2\epsilon^2\eta+\delta\varrho^2)\right)^2} < 0, \frac{d\pi_R^{M*}}{d\eta} = 0.$$

$$\frac{d\pi_M^{R*}}{d\lambda} = \frac{32\beta\delta^2\eta^2(\lambda-\beta\epsilon)B^2}{(2A+H)^2} > 0, \frac{d\pi_R^{R*}}{d\lambda} = \frac{4\beta\delta^2(\lambda-\beta\epsilon)B^2}{(\frac{A+H}{\eta})^2} > 0,$$

$$\frac{d\pi_M^{R*}}{d\delta} = -\frac{16\beta\delta\eta^2(-\beta\epsilon+\lambda)^2B^2}{(2A+H)^2} < 0, \frac{d\pi_R^{R*}}{d\delta} = -\frac{2\beta\delta(-\beta\epsilon+\lambda)^2B}{(\frac{A+H}{\eta}))^2} < 0,$$

$$\frac{d\pi_M^{R*}}{d\eta} = -\frac{8\beta^2\delta^2\eta\varrho^2B}{(2A+H)^2} < 0, \frac{d\pi_R^{R*}}{d\eta} = 0.$$



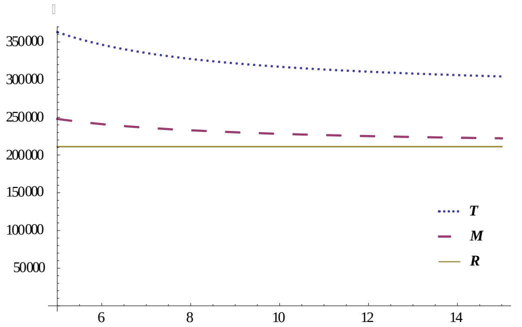


Figure.4. Total profits vary with parameter
λ and $\eta(\phi = 10000, \beta = 30, c = 60, \epsilon = 0.5, \lambda = 16, \delta = 90, \eta = 10, \varrho = 0.3)$

.

The above proposition indicates that the total profits of centralized mode is the highest while the total profits of the retailer recovery mode is the lowest. Manufacturer and retailer's profits of manufacturer recovery mode all are higher than that of the retailer recovery mode. The total profits, manufacturer and retailer's profits all increase with the increase of the coefficient of demand responsiveness to CLSC's own quality, and decrease with the increase of fixed quality cost parameter. In centralized decision model and manufacturer recovery mode, the total profits and retailer's profits both decrease with the increase of the recovery cost parameter while the recovery rate of retailer recovery mode has nothing to do with the parameter. As shown in fig.4, for decentralized supply chain, the manufacture recovery mode is advantageous to increase total profits for the CLSC.

5 SUMMARY

In this paper, by using game theory, we have studied product quality and waste product recovery rate decision problems of CLSC which contains a manufacture and a retailer, the centralized decision model, retailer recovery and manufacturer recovery model are analysed. Research results show that the quality level, recovery rate, demand and profits all are the highest in the centralized decision model while they are higher in the manufacture recovery mode of decentralized decision model. At the same time, for improving the product quality, recovery rate, demand and profits of CLSC, increasing λ, reducing η and δ are an important way.

In this paper, we only consider a closed-loop supply chain composed of a manufacturer and a retailer. In the actual closed-loop supply chain, it is often made up of many manufacturers and retailers, in this case, the quality level and the recovery rate how to decide, and the next step we will be carried out.

6 ACKNOWLEDGEMENTS

This work is supported by the science and technology foundation of Guizhou province (Guizhou J word [2013]2085), Guizhou Natural Science Foundation Project ((2014) 264).

7 REFERENCES

► [1] Chao, G. H., Iravani, S. M. R., Savaskan, R. C., 2009, Quality Improvement Incentives and Product Recall Cost Sharing Contracts, Management Science, 55/7, 1122-1138

► [2] Brekke, K. R., Siciliani, L., Straume, O. R.,2010,Price and quality in spatial competition, Regional Science and Urban Economics,40/6,471-480

► [3] El Ouardighi F., Kim, B., 2010,Supply quality management with wholesale price and revenue-sharing contracts under horizontal competition, European Journal of Operational Research,206/2,329-340

► [4] Xie, G., Wang S., Lai, K. K., 2011, Quality improvement in competing supply chains, International Journal of Production Economics,134/1,262-270

► [5] Xie, G., et al., 2011, Quality investment and price decision in a risk-averse supply chain, European Journal of Operational Research,214/2,403-410

► [6] Lambertini, L., Tampieri A., 2012, Low-quality leadership in a vertically differentiated duopoly with Cournot competition, Economics Letters, 115/3,396-398

► [7] Lee, C. H., Rhee, B. D., Cheng, T. C. E., 2013, Quality uncertainty and quality-compensation contract for supply chain coordination, European Journal of Operational Research,228/3,582-591

► [8] Ma, P., Wang, H., Shang, J., 2013, Contract design for two-stage supply chain coordination: Integrating manufacturer-quality and retailer-marketing efforts, International Journal of Production Economics,146/2,745-755

► [9] Ma, P., Wang, H., Shang, J., 2013, Supply chain channel strategies with quality and marketing effort-dependent demand, International Journal of Production Economics,144/2,572-581

► [10] El Ouardighi, F., 2014, Supply quality management with optimal wholesale price and revenue sharing contracts: A two-stage game approach. International Journal of Production Economics,156/5,260-268

► [11] Yoo, S. H., 2014, Product quality and return policy in a supply chain under risk aversion of a supplier, International Journal of Production Economics, 154/4,146-155

► [12] Jia Pingxie, Lin Nachi, Liang Ling, 2012, Remanufacturing optimal production decision based on the product quality in manufacturing, Journal of Management Sciences in China, 15/8,35-41

► [13] Xiong Zhongkai, Cao Jun, Liu Kejun,2007,Research on closed-loop supply chain recycling quality control based on dynamic game theory, China management science, 15/4,19-26

► [14] Dixit, A., Pindyck, R. S., Sldal S., 1999, A markup interpretation of optimal investment rules. The Economic Journal.109/455,179-89

► [15] Banker, R. D., Khosla I., Sinha, K. K., 1998, Quality and competition, Management Science, 44/9,1179-1192