PERISHABLES' INVENTORY CONTROL MODEL UNDER TIME-VARYING AND CONTINUOUS DEMAND

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ABSTRACT: This paper considers the hysteresis perishable characteristics and shortage amount of delayed rate of perishables comprehensively, and establishes a retailer inventory model shortage point and metamorphism point. Perishables shortage occurred after metamorphism, the retailer inventory cost in addition to the ordering cost, the inventory holding cost, the shortage losses cost involves also the cost of metamorphism. Finally, a numerical example is given to analyze the proposed model and the parameter sensitivity is discussed.

KEY WORDS: perishables, inventory control model, continuous demand, time-varying

1 INTRODUCTION

With the improvement of the level of production modernization, the production scale and inventory scale gradually increasing, the inventory management of perishables has become one of the hot issues which has attracted by scholars in recent decades. Perishables can be also called seasonal products, perishable products and the short life cycle products etc. with the obvious characteristics, such as the short sales cycle, long production lead time, uncertainty demands, low residual value of unsold in end of the period and higher processing cost etc (Liu, Lin, & Chen, 2012). Perishables has characteristics of high time requirement, therefore the research on the hysteresis perishables inventory strategy under time-varying demand has a very important contribution.

Some researches about inventory model considering the deterioration rate have been presented. Moon studied the inventory model with constant deterioration rate (Moon, Giri, & Ko, 2005). Aggarwal established the optimal ordering quantity model of deterioration products where it is assumed that the deterioration rate is constant and the deferred payment is allowed (Aggarwal & Jaggi, 1995). Chu proved the total cost is a piecewise convex function in further and proposed a simpler solution process based on the research by Aggarwal (Chu, Chung, & Lan, 1998). Chang proposed the optimal economic order model that considered the inflation and allowed the deferred payment with the deterioration rate unchanged (Change, 2004). Shah established a bulk purchasing model for deteriorating items under two scenarios, fixed period and unfixed period (Shah & Shah, 1993; Shah, 1998), and a bulk purchasing model with a constant deterioration rate is given where the replenishment lead time is set to zero (Shah & Shah, 2000). Lin established an inventory replenishment policy with the assumption that deterioration rate increases linearly with time and in a fixed period the inventory update speed and the service level are equal (Lin, Tan & Lee, 2000). We established a deterioration products inventory model in which the rate of deterioration obeys two-parameter Weibull distribution (Wee, 1999). Chauhuri introduced the two-parameter and the three-parameter Weibull distribution function to replace linear and exponential related deterioration rate. And the two-parameter and the three-parameter Weibull distribution function makes inventory model more tally with the actual situation (Chakrabarty, Girl & Chauhuri, 1998). Chen (Chen, 1998), Chang (Chang & Dye, 2000) studied and found the deterioration rate of perishable goods was a two-parameter Weibull distribution, considering the service level, replenishment cycle and price discounts of the deterioration products inventory model. Moon and Lee respectively established an inventory replenishment model for perishable goods referring to exponential distribution and Gaussian distribution (Moon & Lee, 2000). Guo studied the optimal economic order model while the deterioration rate of the deterioration products is (Guo, 2004). Huang established an EOQ model for the deterioration products, taking the factors which affecting the storage time on the deterioration rate into accounts (Huang, Huang & Cai, 2006). Li established an optimal ordering and inventory model for the deterioration products based on three-parameter Weibull distribution function (Li, Huang & Luo, 2004). Peng established an optimal inventory and pricing model for the deterioration products with the assumption that deterioration rate

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is an exponential function and considering the time value of capital and the effect of purchase quantity discounts (Peng & Tian, 2004).

The research on backlogging rate of inventory mode has been also studied. Heng established an inventory control model with the hysteresis products supply rate change as the waiting time length of customer and the shortage and ordering costs are different at different cycles (Heng, Labban & linn, 1991). Papachristos researched on backlogging rate which reduced with the customer waiting time from the perspective of manufacturer, and using the deteriorating items inventory model where quantity discounts increase with respect to the sales volume (Papachristos & Skouri, 2007). Chang established the products profit function based on two situations respectively, for example the shortage volume is fully delayed and the shortage volume does not allow delayed (Chang, 2004). Chung considered comprehensively the impact of cash discounts and time value of capital, then established a deterioration products inventory replenishment strategy model (Chung & Lin, 2001). Zhao studied the impact caused by the change of the inventory cost and the inventory level on the selling rate with the purpose of the minimum cost and maximum profit (Zhao & Liu, 2004). Pi studied vendor management optimal inventory policy of deterioration products under the Supply Chain Management environment (Pi, Meng & Huang, 2010). Leng established a nonlinear deterioration products inventory model assuming that during the stockout period, customers' amount of lost is a Gaussian distribution and goods shortage permission (Leng, Liu & Huang, 2004). Yang used the (s,S) inventory policy to restock, established a profit model for deterioration products assuming that the stock level affect the sales rate, shortage completely delayed (Yang & Huang, 2005). Luo established an EOQ model for the deterioration products based on the two situations that shortage completely delayed and no shortage delayed (Luo, Xiong & Yang, 2002).

This paper establishes a new stock control model considering the deterioration rate and backlogging rates. In the proposed model, the amount of the products is not enough to supply once the metamorphism occurs, and the retailer inventory cost in addition to the ordering cost, the inventory holding cost, the cost of shortage losses involves also the cost of metamorphism.

2 VARIABLE DEFINITIONS

The inventory system established is operating in a finite time horizon H.

Retailers' demand per unit time is a linear function as the inventory level, and let $f(t)$ denotes the demand rate at time t is:

$$f(t)=\begin{cases} D+\delta I(t) & I(t)\geq 0 \\ D & I(t)<0 \end{cases} \tag{1}$$

where D is the customers' demand rate at time t, δ is an impact coefficient of the stock volumes on the sale rate, D, δ are constants ($D>0,\ \delta>0$). $I(t)$ is the inventory level at time t assuming that the unit time is very short. This paper deals with the time variable t approximately as a continuous variable (Mi & Zhou, 2010).

The shortage appears when the retailer inventory holdings cannot meet the customer demand. The shortage exits partially in the next cycle, but in the end of cycle it is not allowed, and the shortage volume delayed rate is related to the customer waiting time. Set the proportional function that customer is willing to wait delivery during the shortage $b(t): b(t)=e^{-\beta t}$,

$0<b(t)<1,\ t_0\leq t<T$. β is the scale factor, t_0 is the time length that is not allowed shortage during a cycle, and T is the duration of an order cycle.

Demand rate of retailers waiting for delivery at time t is $S(t)$.

Retailer inventory deterioration rate is constants θ, and $\theta>0$. Hysteresis area where deterioration does not occur is t_h, the latter is metamorphic area (Cao, Che & Wu, 2012).

Retailers take the same cycle restock. Order number is n within the interval H, then the restock cycle is $T=H/n$, and the restock time point is $T_i=(i-1)H/n, T_n=H$. The ordering cost is C_b. Instantaneous replenishment is considered, that is to say the replenishment lead time is 0. The inventory holding cost per unit time and per unit perishables is d, the shortage loss cost per unit time and per unit perishables is f, the deterioration cost per unit time and per unit perishables is g, the total inventory holding cost over a cycle is C_d, the total shortage losses cost is C_f, the total deterioration cost is C_g, and the total inventory cost over the interval H is C_h.

According to the above assumptions, the delayed demand rate is $b(t)$ at time t during shortage $[t_{i,0},T_{i+1}]$. Therefore, the demand of customers waiting $S(t)$ at the shortage time t should satisfy the following differential equation (Wang, 2011):

$$\frac{dS(t)}{dt}=b(t), \quad t_{i,0}\le t\le T_{i+1} \tag{2}$$

The boundary condition is $S(t_{i,0})=0$, and then we have the solution:

$$S(t)=\int_{t_{i,0}}^{t} b(u)du=\frac{e^{-\beta t_0}-e^{-\beta t}}{\beta} \tag{3}$$

3 INVENTORY CONTROL MODE

According to assumption (6), the cycle horizon H is divided into n equal with the length $l=H/n$. The replenishment point at the beginning of i_{th} cycle $T_i=(i-1)l=(i-1)H/n$. The last cycle is not allowed to be out of stock, so $t_n=T_{n+1}=H$. It includes two periods: non-deterioration period $[T_i,t_{i,h})$ and deterioration period $(t_{i,h},T_{i+1}]$ where $t_{i,h}$ is the deterioration point of the i_{th} cycle. Other cycles $[T_i,T_{i+1}]$ include the following periods: when the shortage point $t_{i,0}$ occurs in the non-deterioration of the $t_{i,h}$, namely when shortage point is the former. It is divided into inventory holding and non-deterioration period $[T_i,t_{i,0})$ and shortage period $[t_{i,0},T_{i+1}]$. The out of stock point $t_{i,0}$ occurs in the deterioration period $t_{i,h}$ which implies that when the deterioration point is the former. It is divided into inventory holding and non-deterioration period $[T_i,t_{i,h})$, inventory holding and deterioration period $(t_{i,h},t_{i,0})$ and out of stock period $[t_{i,0},T_{i+1}]$. $t_{i,0}$ is the out of stock point of the i_{th} cycle. Since $\delta>0$, it is known that the demand increases gradually with the stock volumes diminished rapidly over one cycle, thus the inventory function within a cycle is a downwardly convex decreasing curve. $I_i(t)$ denotes that the inventory level of retailer at t during i_{th} replenishment cycle.

3.1 Inventory control model with shortage point previous (n-1) cycles

The retailer is not allowed to be shortage state in the last cycle as mentioned in the previous section, so we only discusses the first $(n-1)$ cycles. It includes two periods: inventory holding without deterioration period $[T_i,t_{i,0})$ and shortage period $[t_{i,0},T_{i+1}]$.

In the interval $[T_i,t_{i,0})$, the inventory items do not turn deteriorative and the inventory level is positive. The change rate of inventory level is equal to the demand rate of the items when $I(t)\ge 0$. So with in $[T_i,t_{i,0})$, the instantaneous state of inventory level should satisfy the following condition:

$$\frac{dI_{i,1}(t)}{dt}=-D-\delta I_{i,1}(t), \quad T_i\le t<t_{i,0}$$

(4)

The boundary condition is $I_{i,1}(t_{i,0})=0$, the inventory of retailer at time t is:

$$I_{i,1}(t)=\frac{D}{\delta}[e^{\delta(t_{i,0}-t)}-1] \quad [T_i,t_{i,0}) \tag{5}$$

In the interval $(t_{i,0},T_{i+1}]$, the inventory level is negative and there is no deterioration. The change rate of inventory level is equal to the demand rate of the items when $I(t)<0$. So during the interval $(t_{i,0},T_{i+1}]$, the instantaneous state of inventory level should satisfy the following condition:

$$\frac{dI_{i,2}(t)}{dt}=-D, \quad t_{i,0}<t\le T_{i+1} \tag{6}$$

The boundary condition is $I_{i,2}(t_{i,0})=0$, the inventory of retailer at t is:

$$I_{i,2}(t)=D(t_{i,0}-t) \quad (t_{i,0},T_{i+1}] \tag{7}$$

The ordering cost of the retailer in the i_{th} cycle is C_b. The total inventory holding cost of the retailer over a cycle is defined as:

$$C_d=d\int_{T_i}^{t_{i,0}} I_{i,1}(t)dt=d\frac{D}{\delta^2}(e^{\delta t_{i,0}}-\delta t_{i,0}-1) \tag{8}$$

The shortage cost of the retailer in the i_{th} cycle is given by:

$$C_f = f\int_{t_0}^{T} |S(t)|dt = \frac{fT}{\beta}e^{-\beta t_0} - \frac{ft_0}{\beta}e^{-\beta t_0} + \frac{fe^{-\beta T}}{\beta^2} - \frac{fe^{-\beta t_0}}{\beta^2} \quad (9)$$

The inventory cost of the retailer over a finite horizon H (in addition to the last cycle) is:

$$C_h = (n-1)C_b + \sum_{i=1}^{n-1}(C_d + C_f) \quad (10)$$

Optimal solution analysis: from Eq. (10), the total inventory cost C_h is the continuous function of the independent variable $t_{i,0}$. By determining the value of $t_{i,0}$, the objective function C_h can be obtained to the minimum, which corresponds to obtain the maximum value of $(C_d + C_f)$. The necessary conditions for $(C_d + C_f)$ to the maximum are given as:

$$\frac{d(C_d + C_f)}{dt_{i,0}} = 0 \quad (11)$$

$$e^{-\beta t_{i,0}} f(t_{i,0} - T_{i+1}) + dDe^{\delta t_{i,0}} - \frac{dD}{\delta} = 0 \quad (12)$$

Proposition 1: Eq. (12) has the only solution.

Proof:

Firstly, in order to prove the existence of zero solution of Eq. (12), let:

$$g(t_{i,0}) = e^{-\beta t_{i,0}} f(t_{i,0} - T_{i+1}) + dDe^{\delta t_{i,0}} - \frac{dD}{\delta} \quad (13)$$

Then:

$$\lim_{t_{i,0}\to 0} g(t_{i,0}) = -T_{i+1}f + dD - \frac{dD}{\delta} < 0 \quad (14)$$

$$\lim_{t_{i,0}\to\infty} g(t_{i,0}) = \lim_{t_{i,0}\to\infty}[e^{-\beta t_{i,0}} f(t_{i,0} - T_{i+1}) + dDe^{\delta t_{i,0}} - \frac{dD}{\delta}] > 0 \quad (15)$$

So: Eq. (12) must has a zero solution.

In the following the uniqueness is proved, and taking the derivative of the function $g(t_{i,0})$ and $t_{i,0}$.

$$\frac{dg(t_{i,0})}{dt_{i,0}} = \beta e^{-\beta t_{i,0}} f(T_{i+1} - t_{i,0}) + e^{-\beta t_{i,0}} f + \delta dDe^{\delta t_{i,0}} > 0 \quad (16)$$

Thus: $g(t_{i,0})$ is the increasing function of the independent variable $t^*_{i,0}$, so Eq. (12) has the only solution.

Proof completed.

Proposition 2: That $t_{i,0}$ meets Eq. (12) is the unique solution to obtain the minimum of $(C_d + C_f)$.

Proof:

$$\frac{d^2(C_d + C_f)}{dt^2_{i,0}} = e^{-\beta t_{i,0}}[\beta f(T_{i+1} - t_{i,0}) + f] + \delta dDe^{\delta t_{i,0}} > 0 \quad (17)$$

So: $(C_d + C_f)$ has the minimum at $t^*_{i,0}$.

Proof completed.

3.2 Inventory control model with metamorphism point previous (n-1) cycles

The retailer is not allowed to be shortage in the last cycle, so this section discusses the first $(n-1)$ cycles separated from the n_{th} cycle.

1) The inventory level of retailer in the first $(n-1)$ cycles

a) Within the interval , the inventory items do not turn deteriorative and inventory level is positive. The change rate of inventory level is equal to the demand rate of the items when $I(t) \geq 0$.

So within $[T_i, t_{i,h}]$, the instantaneous state of inventory level should satisfy:

$$\frac{dI_{i,3}(t)}{dt} = -D - \delta I_{i,3}(t), \quad T_i \leq t \leq t_{i,h} \quad (18)$$

$I(t)$ changes continuously.

So:

$$I_{i,3}(t_{i,h}) = \frac{D}{\theta+\delta}\left[e^{(\theta+\delta)(t_{i,0}-t_{i,h})} - 1\right] \quad (19)$$

The inventory of retailers at t is:

$$I_{i,3}(t) = e^{\delta(t_{i,h}-t)}[\frac{D}{\delta} + \frac{D}{\theta+\delta}(e^{(\theta+\delta)(t_{i,0}-t_{i,h})} - 1)] - \frac{D}{\delta} \quad [T_i, t_{i,h}] \quad (20)$$

b) Within the interval $(t_{i,h}, t_{i,0})$, inventory items turn deteriorative and inventory level is positive. The change rate of inventory level is equal to the demand rate and the deterioration rate of the items when $I(t) \geq 0$.

So within $(t_{i,h}, t_{i,0})$, the instantaneous state of inventory level should satisfy the following condition:

$$\frac{dI_{i,4}(t)}{dt} = -D - \delta I_{i,4}(t) - \theta I_{i,4}(t), \quad t_{i,h} < t < t_{i,0} \tag{21}$$

The boundary condition is $I_{i,4}(t_{i,0}) = 0$, and the inventory of retailer at t is:

$$I_{i,4}(t) = \frac{D}{\theta+\delta}[e^{(\theta+\delta)(t_{i,0}-t)} - 1] \quad (t_{i,h}, t_{i,0}) \tag{22}$$

c) Within the interval $[t_{i,0}, T_{i+1}]$, the inventory level is negative. The change rate of inventory level is equal to the demand rate of the items when $I(t) < 0$.

So within the time duration $[t_{i,0}, T_{i+1}]$, the instantaneous state of inventory level should satisfy the following condition:

$$\frac{dI_{i,5}(t)}{dt} = -D, \quad t_{i,0} \leq t \leq T_{i+1} \tag{23}$$

The boundary condition is $I_{i,5}(t_{i,0}) = 0$, and the inventory of retailer at t is:

$$I_{i,5}(t) = D（t_{i,0} - t） \quad [t_{i,0}, T_{i+1}] \tag{24}$$

The ordering cost of the retailer in the i_{th} cycle is C_b.

The total inventory holding cost of the retailer in the i_{th} cycle is as follows:

$$C_d = d[\int_{T_i}^{t_{i,h}} I_{i,3}(t)dt + \int_{t_{i,h}}^{t_{i,0}} I_{i,4}(t)dt]$$

$$= d[\frac{D}{\delta} + \frac{D}{\theta+\delta}(e^{(\theta+\delta)(t_{i,0}-t_{i,h})} - 1)]\frac{e^{\delta t_{i,h}} - 1}{\delta} -$$

$$d\frac{Dt_{i,h}}{\delta} + \frac{dD}{(\theta+\delta)}(\frac{e^{(\theta+\delta)(t_{i,0}-t_{i,h})} - 1}{\theta+\delta} - t_{i,0} + t_{i,h}) \tag{25}$$

The shortage cost of the retailer in the i_{th} cycle is:

$$C_f = f\int_{t_{i,0}}^{T_{i+1}} |S(t)| dt = \frac{fT_{i+1}}{\beta} e^{-\beta t_{i,0}}$$

$$-\frac{ft_{i,0}}{\beta} e^{-\beta t_{i,0}} + \frac{fe^{-\beta T_{i+1}}}{\beta^2} - \frac{fe^{-\beta t_{i,0}}}{\beta^2} \tag{26}$$

The deterioration cost of the retailer in the i_{th} cycle is:

$$C_g = g\int_{t_{i,h}}^{t_{i,0}} I_{i,4}(t)dt$$

$$= \frac{gD}{\theta+\delta}\left(\frac{e^{(\theta+\delta)(t_{i,0}-t_{i,h})} - 1}{\theta+\delta} - t_{i,0} + t_{i,h}\right) \tag{27}$$

The inventory cost of the retailer over the time horizon H (in addition to the last cycle) is:

$$C_h = (n-1)C_b + \sum_{i=1}^{n-1}(C_d + C_f + C_g) \tag{28}$$

2) The inventory level of retailer in the n_{th} cycle

a) Within the interval $[T_n, t_{n,h})$, the inventory items do not turn deteriorative. The change rate of inventory level is equal to the demand rate of the items when $I(t) \geq 0$.

So within $[T_n, t_{n,h})$, the instantaneous state of inventory level should satisfy the following differential equation:

$$\frac{dI_{n,1}(t)}{dt} = -D - \delta I_{n,1}(t), \quad T_n \leq t < t_{n,h} \tag{29}$$

$I_{n,1}(t)$ is continuous, so the inventory of retailers at t is:

$$I_{n,1}(t) = \left\{\frac{D}{(\theta+\delta)}[e^{(\theta+\delta)(T_{n+1}-t_h)} - 1] + \frac{D}{\delta}\right\} e^{\delta(t_h - t)}$$

$$-\frac{D}{\delta} \quad [T_n, t_{n,h}) \tag{30}$$

b) During the time range $(t_{n,h}, T_{n+1}]$, the inventory items turn to deterioration. The change rate of inventory level is equal to the demand rate and the deterioration rate of the items when $I(t) \geq 0$.

So within the interval $(t_{n,h}, T_{n+1}]$, the instantaneous state of inventory level should satisfy the following differential equation:

$$\frac{dI_{n,2}(t)}{dt} = -D - \delta I_{n,2}(t) - \theta I_{n,2}(t), \quad t_{n,h} < t \leq T_{n+1} \tag{31}$$

The boundary condition is $I_{n,2}(T_{n+1})=0$, the inventory of retailer at t is:

$$I_{n,2}(t)=\frac{D}{(\theta+\delta)}[e^{(\theta+\delta)(T_{n+1}-t)}-1] \quad (t_{n,h},T_{n+1}] \tag{32}$$

The ordering cost of the retailer in the last cycle is C_b.

The total inventory holding cost of the retailer in the last cycle is as follows:

$$C_d=d[\int_{T_n}^{t_{n,h}} I_{n,1}(t)dt+\int_{t_{n,h}}^{T_{n+1}} I_{n,2}(t)dt]$$

$$=d[\frac{\left\{\frac{D}{(\theta+\delta)}[e^{(\theta+\delta)(T_{n+1}-t_h)}-1]+\frac{D}{\delta}\right\}}{\delta}(e^{\delta t_{n,h}}-1)-\frac{Dt_{n,h}}{\delta}+$$

$$\frac{D}{\theta+\delta}(\frac{e^{(\theta+\delta)(T_{n+1}-t_{n,h})}-1}{\theta+\delta}-T_{n+1}+t_{n,h})] \tag{33}$$

The deterioration cost of the retailer in the last cycle is:

$$C_g=g\int_{t_{n,h}}^{T_{n+1}} I_{n,2}(t)dt$$

$$=\frac{gD}{\theta+\delta}[\frac{e^{(\theta+\delta)(T_{n+1}-t_{n,h})}-1}{\theta+\delta}-T_{n+1}+t_{n,h}] \tag{34}$$

The inventory cost of the retailer in the last cycle is:

$$C_t=C_b+C_d+C_g \tag{35}$$

Optimal solution analysis: it shows that the total inventory cost C_t is the continuous function of independent variable T_{n+1}. By determining the value of T_{n+1}, the objective function C_t is obtained to the minimum, and the necessary condition is shown as:

$$\frac{dC_t}{dT_{n+1}}=\frac{(g+d)D}{\theta+\delta}[\frac{e^{-(\theta+\delta)(T_{n+1}-t_{n,h})}(-)(\theta+\delta)}{\theta+\delta}-1]$$

$$=\frac{-(g+d)D}{\theta+\delta}[e^{-(\theta+\delta)(T_{n+1}-t_{n,h})}+1] \tag{36}$$

By $\frac{dC_t}{dT_{n+1}}<0$, the function C_t is a monotone decreasing function. Since C_t is a bounded function, there is T^*_{n+1} to make C_t to the minimum.

Proof completed.

4 EMPIRICAL ANALYSIS

4.1 The setting of the control parameters

The paper investigated perishable productions of a supermarket in Handan as long as possible; the specific parameters are shown in Table 1.

Table 1. The Parameters Used in the Proposed Model

The symbols and meanings of the parameters	Parameter value
Planning period H	90 days
The length of an order cycle T	15 days
Order times n in planning period H	6 times
Customer demand rate per unit time D	600 articles
The coefficient of inventory impacting sales δ	0.15
The ratio willing to wait for delivery of customers β	0.2
Deteriorating rate θ	0.35
Order cost of retailer each time C_b	200 RMB/time
Inventory holding cost per unit time per unit product d	0.2
Inventory shortage cost per unit time per unit product f	0.8
Deterioration cost per unit time per unit product g	0.6
Time length without deterioration in a cycle $t_{i,h}$	13 days

4.2 Inventory control model analysis

1) Solve the given initial value

The paper uses Matlab to calculate the numerical example, and obtains the previous deterioration point, the optimal shortage point of shortage time in deterioration period is 13.1 days, and the retailer's optimal inventory cost is 168100 RMB.

2) Sensitivity analysis of parameters

a) The influence caused by the change of the customer demand rate per unit time on the total inventory cost

Keeping other parameters unchanged, the influence caused by the change of in the customer demand rate per unit time on the optimal shortage point and the total inventory cost is analyzed. The results are shown in Table 2.

Table 2. Sensitivity Analysis of the Customer Demand Rate D Per Unit Time

D	580	590	610	620
t_0^* (day)	13.1	13.1	13.1	13.1
C^* (RMB)	162530	165320	170880	173660

Table 2 shows that in the state of previous product deterioration point, the influence caused by the change of the customers demand rate D per unit time on the optimal shortage point and the total inventory cost. With the increase of the customer demand rate D, the optimal shortage point basically remains unchanged and the inventory cost gradually increases. The change of the customers demand rate D per unit time has a less influence on the optimal shortage point and the inventory cost.

b) The influence caused by the change of the inventory affecting sale rate δ on the inventory cost

Keep other parameters unchanged, the influence caused by the coefficient of the inventory impacting sale rate on the optimal shortage point and inventory cost is studied. The results are given in Table 3.

Table 3. Sensitivity Analysis of the Coefficient of the Inventory Impacting Sale Rate δ

δ	0.13	0.14	0.16	0.17
t_0^* (day)	13.122	13.111	13.091	13.083
C^*(RMB)	149650	158500	178610	190070

Table 3 shows that in the state of previous product deterioration point, the influence caused by the coefficient of inventory impacting sale rate δ on the optimal shortage point and the total inventory cost. With the increase of the coefficient of inventory impacting sale rate δ, the optimal shortage point gradually decreases, and the inventory cost gradually increases. The change of the coefficient of inventory impacting sales rate δ has a less influence on the optimal shortage point and the inventory cost.

c) The influence caused by the change of customer's willing to wait for delivery rate β on the inventory cost

Keep other parameters unchanged, analysis on the influence caused by the coefficient of customer's willing to wait for delivery rate on the optimal shortage point and inventory cost are done. The calculation results are shown in Table 4.

Table 4. Sensitivity Analysis of Customer's Willing to Wait for Delivery Rate β

β	0.18	0.19	0.21	0.22
t_0^* (day)	13.1	13.1	13.1	13.1
C^* (RMB)	153090	153090	153090	153090

Table 4 shows that in the state of previous product deterioration point, the influence caused by the coefficient of customer s' willing to wait for delivery rate β on the optimal shortage point and the total inventory cost. With the increase of the coefficient of customers' willing to wait for delivery rate β, the optimal shortage point and the inventory cost basically remains unchanged. The change of the coefficient of customer's willing to wait for delivery rate β has a less influence on the optimal shortage point and the inventory cost.

d) The influence caused by the change of the deterioration rate θ on the inventory cost

Keep other parameters unchanged, analysis the influence caused by the changed deterioration rate on the optimal shortage point and inventory cost are discussed. The calculation results are shown in Table 5.

Table 5. Sensitivity Analysis of the Deterioration Rate

θ	0.33	0.34	0.36	0.37
t_0^* (day)	13.106	13.103	13.098	13.095
C^* (RMB)	167950	168020	168270	168310

Table 5 shows that in the state of previous product deterioration point, the influence caused by the deterioration rate θ on the optimal shortage point and the total inventory cost. With the increase of the deterioration rate θ, the optimal shortage point gradually decreases, the inventory cost gradually increases. The change of the deterioration rate θ has a less influence on the optimal shortage point and the inventory cost.

e) The influence caused by the change of inventory holding cost d on the inventory cost

Keep other parameters unchanged, analysis of the influence caused by the changed inventory holding cost on the optimal shortage point and inventory cost are discussed. The calculation results are as shown in Table 6.

Table 6. Sensitivity Analysis of the Inventory Holding Cost d

d	0.18	0.19	0.21	0.22
t_0^* (day)	13.108	13.104	13.098	13.095
C^* (RMB)	138350	145730	160520	167910

Table 6 shows that in the state of previous product deterioration point, the influence caused by the holding cost d on the optimal shortage point and the total inventory cost. With the increase of the holding cost d, the optimal shortage point gradually decreases, the inventory cost gradually increases. The change of the holding cost d has a less influence on the optimal shortage point and the inventory cost.

f) The influence caused by the change of inventory shortage cost f on the inventory cost

Keep other parameters unchanged, the influence caused by the changed inventory shortage cost on the optimal shortage point and inventory cost are given. The calculation results are shown in Table 7.

Table 7. Sensitivity Analysis of the Inventory Shortage Cost f

f	0.78	0.79	0.81	0.82
t_0^* (day)	13.1	13.1	13.1	13.1
C^* (RMB)	153090	153090	153090	153090

Table 7 shows that in the state of previous product deterioration point, the influence caused by the inventory shortage cost f on the optimal shortage point and the total inventory cost. With the increase of the inventory shortage cost f, the optimal shortage point and the inventory cost remain unchanged. The change of the inventory shortage cost f has a less influence on the optimal shortage point and the inventory cost.

g) The influence caused by the change of deterioration cost g

Keep other parameters unchanged, the influence caused by the changed deterioration cost on the optimal shortage point and inventory cost is analyzed finally. The calculation results are shown in Table 8.

Table 8. Sensitivity Analysis of the Deterioration Cost g

g	0.58	0.59	0.61	0.62
t_0^* (day)	13.099	13.099	13.102	13.103
C^* (RMB)	153000	153030	153200	153260

Table 8 shows that in the state of previous product deterioration point, the influence caused by the deterioration cost g on the optimal shortage point and the total inventory cost. With the increase of the deterioration cost g, the optimal shortage point and the inventory cost increase. The change of the deterioration cost g has a less influence on the optimal shortage point and the inventory cost.

5 CONCLUSIONS

Through the parameters sensitivity analysis, it shows that the customer demand rate D per unit time and the coefficient of inventory impacting sale rate δ have a larger effect on the inventory cost. Customer's willing to wait for delivery rate β and the deterioration rate θ have a less effect on the inventory cost. In the practical applications, with the change of parameters, retailers should pay attention to the influence caused by D and δ on the inventory cost in order to adjust inventory strategy timely. The further research focuses on the inventory control model with discrete random variable considering the shortage point and metamorphism point.

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