

# NUMERICAL SIMULATION OF REFINING ROLLING PROCESS BASED ON ADAPTIVE BEM PARALLEL ALGORITHM

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**ABSTRACT:** First, the paper constructs three-dimensional elastoplastic frictional contact adaptive boundary element method and respectively introduces boundary element space and two solid state of contact, then proposes three-dimensional elastoplastic BEM. Second, the paper constructs two adaptive control technology models. Finally the paper designs parallel condensed algorithm. In the algorithm, we combine with three algorithms and do numerical simulation. So we can reduce the system steady-state error, reduce the system overshoot amount, improve the system response speed and improve calculation accuracy and computational efficiency.

**KEY WORDS:** Refined rolling; Numerical simulation; Adaptive control technology; BEM; Parallel algorithm

## 1 INTRODUCTION

Rolling problems is mainly the study of elastic-plastic contact problem of roller and rolled piece. The calculation precision of rolling force parameters not only depends on the problem solving method for the simulation precision of the original problem, but also depends on the accuracy of the method itself. So choosing a high precision problem solving method for simulating rolling process is very important. In recent years, numerical solution gets rapid development, finite element method is successfully applied in rolling field, and rolling theory and technology have new progress.

The boundary element method is a powerful tool in computational mechanics and computational mathematics. Compared with FEM[1], due to the dimension reduction, construction of BEM discrete grid is relatively simple. However, from the viewpoint of improving computational efficiency and preventing human subjective errors, we still need to develop the method which can automatically generate optimized discrete grid. The basic idea of adaptive boundary element method is to use computer to automatically determine and improve the accuracy of BEM solution. Obviously, the addition of the adaptive process is of great significance.

In the paper, the rolling process is seen as united whole of roll and rolling piece and we use frictional contact elastoplastic boundary element method to simulate the rolling process. At the same time the paper studies parallel computing method. Then we use adaptive BEM parallel algorithm to numerically simulate refining rolling process. This avoids the disadvantage of usual practice of splitting rolled piece and roll. So we have achieved good results with the fewest number of assumptions.

## 2 THREE - DIMENSIONAL ELASTOPLASTIC FRICTIONAL CONTACT ADAPTIVE BEM

### 2.1. Boundary element space of three-dimensional bem

The boundary element method is a numerical method to solve the boundary integral equation. Contact between two objects occurs on its borders. Proof of existence and uniqueness of BEM solution of frictional contact problem is beginning from boundary element space after solid boundary discretization.

#### 2.1.1. Three-dimensional solid discrete boundary element

We have several methods to get the numerical solution by the discretization of the boundary integral equation[2], for example, on the

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engineering calculation using the configuration method or collocation method. No matter what discrete method you use, you need to discrete boundary, including discrete boundary function. The description of the boundary element has two parts: description of boundary element geometry and boundary function interpolation approximate.

Let boundary  $\Gamma$  be surface in three-dimensional space [3]. We construct approximate surface  $\Gamma_h$  of  $\Gamma$  and give geometrical description of boundary element and piecewise interpolation expression of boundary function. Assume  $\Gamma$  is a combination of piecewise smooth closed surface  $\Gamma_j (j=1,2,\dots,J)$

$$\Gamma = \bigcup_{i=1}^J \Gamma_j, \Gamma_i \cap \Gamma_j \quad (1)$$

For each piece surface  $\Gamma_j$ , there is a two sides of single value smooth mapping  $\varphi_j$ , which maps  $\Gamma_j$  to a bounded closed region in the plane. The inverse mapping is  $\Phi_j = \varphi_j^{-1}$ , so

$$D_j = \varphi_j(\Gamma_j), \Gamma_j = \Phi_j(D_j) \quad (2)$$

Then, assume  $D_j$  is a polygon. Thus we similarly use finite element approximation technique:  $D_j$  is covered with regular triangle section  $T_{hj}$  and then define interpolation function  $\Phi_{jh}$  of mapping  $\Phi_j$  on the each element of subdivision  $T_{hj}$ .  $\Phi_{jh} : D_j \subset R^2 \rightarrow R^3$ , then we get a surface  $\Gamma_{jh}$  in space and

$$\Gamma_{jh} = \Phi_{jh}(D_j) \quad (3)$$

$\Gamma_{jh}$  is the approximate surface of  $\Gamma_j$ , see Fig. (1). The element on  $T_{hj}$  is in the role of mapping  $\Phi_{jh}$  and then we get element on the  $\Gamma_{jh}$ , that is boundary element:

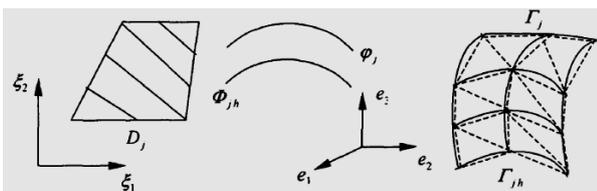


Fig 1. Approximate surface.

## 2.2. Two solid contact

Contact between solid[4], there are two states in the contact surface: fixed and increase with the increase of external load.

### 2.2.1. Two solid contact

Define contact area

$$\Gamma_c = \partial\Omega' \cap \partial\Omega'' \quad (4)$$

Decompose  $\partial\Omega'$  and  $\partial\Omega''$ , see Fig. (2).

$$\begin{cases} \partial\Omega' = \Gamma_u \cup \Gamma_r' \cup \Gamma_k \\ \partial\Omega'' = \Gamma_u \cup \Gamma_r'' \cup \Gamma_k \end{cases} \quad (5)$$

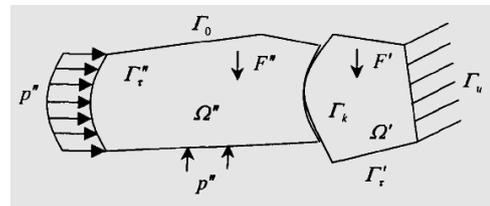


Fig 2. Contact area.

In the Fig. (2),  $\Gamma_u$ ,  $\Gamma_r'$ ,  $\Gamma_r''$  and  $\Gamma_0$  are the boundary disjoint part respectively. Assume  $\Gamma_u$  and  $\Gamma_k$  have positive measure.  $\Gamma_r'$ ,  $\Gamma_r''$  and  $\Gamma_0$  have positive measure or zero measure. When contacting, according to the continuous displacement and surface force balance condition, the following relationship has been established on  $\Gamma_k$

$$\begin{cases} u_n' + u_n'' \leq 0 \\ T_n' = T_n'' \leq 0 \\ (u_n' + u_n'')T_n' = 0 \\ T_t' = T_t'' = 0 \end{cases} \quad (6)$$

where  $u_n' = u_t' \cdot n_t'$ ,  $u_n'' = u_t'' \cdot n_t''$  and  $n' = -n''$ .

### 2.2.1. Increased contact

In the solid deformation process, the contact area may increase[5], see Fig. (3).

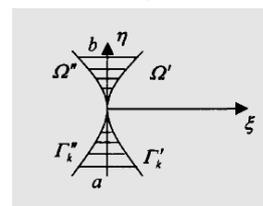


Fig 3. Increased contact area.

Take local coordinate  $(\xi, \eta)$ ,  $\xi$  axis is direction of  $n''$ ,  $\eta$  axis is tangential direction.

Assume

$$\begin{cases} \Gamma'_k = \{(\xi, \eta) | a \leq \eta \leq b, \xi = f'(\eta)\} \\ \Gamma''_k = \{(\xi, \eta) | a \leq \eta \leq b, \xi = f''(\eta)\} \end{cases} \quad (7)$$

where  $f'(\eta)$  and  $f''(\eta)$  are continuous function on  $[a, b]$ . So

$$u''_{\xi} - u'_{\xi} \leq \varepsilon(\eta), \forall \eta \in [a, b] \quad (8)$$

where  $\varepsilon(\eta) = f'(\eta) - f''(\eta)$  is the distance between two objects before deformation,  $u''_{\xi}$  and  $u'_{\xi}$  are components of the displacement vector in the direction of  $\xi$ . So we get

$$\begin{cases} -\frac{T'_{\xi}}{\cos \alpha'} = \frac{T''_{\xi}}{\cos \alpha''} \leq 0 \\ T'_{\eta} = T''_{\eta} = 0 \\ (u''_{\xi} - u'_{\xi} - \varepsilon)T''_{\xi} = 0 \end{cases} \quad (9)$$

$$\begin{cases} \sec \alpha' = \left[ 1 + \left( \frac{df'}{d\eta} \right)^2 \right]^{\frac{1}{2}} \\ \sec \alpha'' = \left[ 1 + \left( \frac{df''}{d\eta} \right)^2 \right]^{\frac{1}{2}} \end{cases} \quad (10)$$

where  $\alpha'$  is the angle between  $\eta$  axis and tangential direction of  $\Gamma'_k$ ,  $\alpha''$  is the angle between  $\eta$  axis and tangential direction of  $\Gamma''_k$ .

### 2.3. Three-dimensional elastoplastic beam

For the three-dimensional problem, the fundamental solution of mechanical displacement equation is as follows[6]:

$$u_{ij}^*(p, q) = \frac{1}{16\pi G(1-\nu)} \left[ (3-4\nu)\delta_{ij} + r_i r_j \right] \quad (11)$$

$$t_{ij}^*(p, q) = \frac{-1}{8\pi(1-\nu)r^2} \left\{ \frac{\partial r}{\partial n} \left[ (1-2\nu)\delta_{ij} + 3r_i r_j \right] - (1-2\nu)(r_i n_j - r_j n_i) \right\} \quad (12)$$

where  $r$  is the distance from source to field point,  $r_i$  is partial derivative of coordinate  $i$ ,  $n$  is normal direction of boundary. Assume boundary of elastomer is divided into  $M$  boundary unit and each boundary unit has  $L$  nodes. After the discrete processing of boundary integral equation we get

$$\begin{aligned} c_{ij} u_j(p) = & \sum_{m=1}^M \sum_{l=1}^L \left( \int_{S_m^e} N_l^{(L)}(\xi, \eta) u_{ij}^*(\xi, \eta, p) J(\xi, \eta) dS^e(\xi, \eta) \right) t_j(q_l^{(m)}) \\ & - \sum_{m=1}^M \sum_{l=1}^L \left( \int_{S_m^e} N_l^{(L)}(\xi, \eta) u_{ij}^*(\xi, \eta, p) J(\xi, \eta) dS^e(\xi, \eta) \right) u_j(q_l^{(m)}) \end{aligned} \quad (13)$$

where  $t_j(q_l^{(m)})$  is surface force value of  $m$  unit's  $l$  node and  $u_j(q_l^{(m)})$  is component of displacement value along the  $j$  direction. We use Gauss integral to deal with the integral of formula (13) and get

$$\int_{S_m^e} N_l^{(L)}(\xi, \eta) u_{ij}^*(\xi, \eta, p) J(\xi, \eta) dS^e(\xi, \eta) \approx \sum_{\alpha=1}^{G1} \sum_{\beta=2}^{G2} N_l^{(L)}(\xi_{\alpha}, \eta_{\beta}) u_{ij}^*(\xi_{\alpha}, \eta_{\beta}, p) J(\xi_{\alpha}, \eta_{\beta}) w_{\alpha} w_{\beta} \quad (14)$$

$$\int_{S_m^e} N_l^{(L)}(\xi, \eta) t_{ij}^*(\xi, \eta, p) J(\xi, \eta) dS^e(\xi, \eta) \approx \sum_{\alpha=1}^{G1} \sum_{\beta=2}^{G2} N_l^{(L)}(\xi_{\alpha}, \eta_{\beta}) t_{ij}^*(\xi_{\alpha}, \eta_{\beta}, p) J(\xi_{\alpha}, \eta_{\beta}) w_{\alpha} w_{\beta} \quad (15)$$

where  $G1$  and  $G2$  are respectively Gauss integral points' number of two directions. Thus, after numerical integration, we obtain system equation set[7]:

$$HU = GT \quad (16)$$

where  $U$  and  $T$  are respectively node displacement vector and cell surface force vector,  $H$  and  $G$  are coefficient matrix. Next we rearrange equation. We move unknown quantity to equation left and known quantity to equation right. At the same time we multiply known quantity and the right coefficient matrix. Then we get

$$AX = F \quad (17)$$

where  $A$  is coefficient matrix associated with unknown quantity and is an asymmetric full matrix,  $X$  is unknown column vector and  $F$  is column vector after multiplying known quantity and the right coefficient matrix. We solve equations (11-17) and then get all unknown quantities on the boundary.

### 2.4. Adaptive control technology computing method

Calculation of adaptive model coefficient has two cases: calculation of current rolled steel coil coefficient and calculation of archived model coefficient. The two calculation methods are the following[8]:

Adaptive coefficient of current rolled steel coil ( $B'$ ):

$$B' = \frac{P_A}{P_C} \quad (18)$$

where  $P_A$  is the actual parameter ( for example rolling force ) and  $P_C$  is setting calculation parameter.

$$C' = (1 - K) \cdot C_1 + K \cdot C_B \quad (19)$$

where  $K$  is experiment constant coefficient,  $C_1$  is Historical accumulation coefficient and  $C_B$  is current calculation coefficient.

In the high speed rolling process, adaptive model is not a very stable process and there will be certain speed fluctuation and elastoplastic change. So in the adaptive control, how to effectively control the change under high speed steady state is the key process of strip rolling.

2.4.1. Control the change under high speed steady state

Considering the importance of the model parameters and its direct impact on product quality, in the adaptive process, computer judges the speed of strip rolling process and develop principle of carrying out adaptive calculation at the maximum speed. So it can better reduce the errors and defects of rolling process[9]. See Fig. (4).

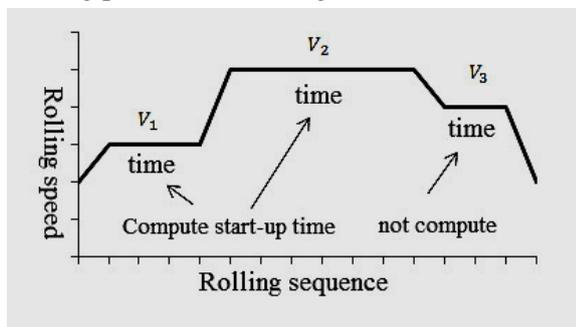


Fig 4. Adaptive calculation in the strip rolling process.

The logic of judgment is  $V = \text{Max}\{V_1, V_2 \dots V_i\}$ . We take the maximum speed value under every steady state in the strip rolling process:  $V_j \geq V$ .

Then we compare the current steady state speed with the maximum speed. If the current steady state speed is more than maximum speed, we start the adaptive calculation; Otherwise, the calculation does not start. So we can obtain the best data under steady state.

2.5 Parallel condensation algorithm design

The paper uses a master-slave programming mode[10]. Control program in the programming mode is called primary process. Control program is responsible for the generation of process, initialization, collection and output results. The rest is from process, performing the actual calculation. Each microcomputer is equipped with a from process and primary process is only on one of the microcomputer. Solving a problem is by the primary process and from process. Generation, management of process, communication between each other such as parallel process are realized through calling the PVM library function. Master-slave programming model is suitable for coarse-grained parallel computing. So in the design algorithm, the division of parallel granularity can not too be thin and try to overlap communication and computation.

In the contact problem of rolled piece and roll, contact area accounts for only a small portion of the entire roll system. The condensation process of reserving contact area influence coefficient occupies 80% of the total serial algorithm computing time. And the condensation process of coefficient matrix of roll and rolling piece are successively completed independently. It has a strong parallelism. In order to solve this contradiction, we developed a parallel condensation algorithm: after first forming roll coefficient matrix, it starts the from process to condense; at the same time in the primary process it forms the coefficient matrix of rolled piece and condenses; finally it returns the condensed result of coefficient matrix of the roll through the network and calculates displacement and stress and outputs the result. Programming ideas and processes of master-slave see Fig. (5).

3 NUMERICAL SIMULATION

Because of the symmetry, we see the quarter of rolled piece as a numerical model, see Fig. (6). The input data and calculation conditions see Table 1. In the beginning, the state of rolling process is not stable. When the materials pass the gap between two rolls for rolling, state begins to become stable. Because of the elastic-plastic deformation, we need to calculate domain integral of rolled piece. We divide rolled piece into four hundred eighty eight-node units, see Fig. (7). For the contact problem, we divide working roll into six hundred twenty-eight four-node boundary elements [12].

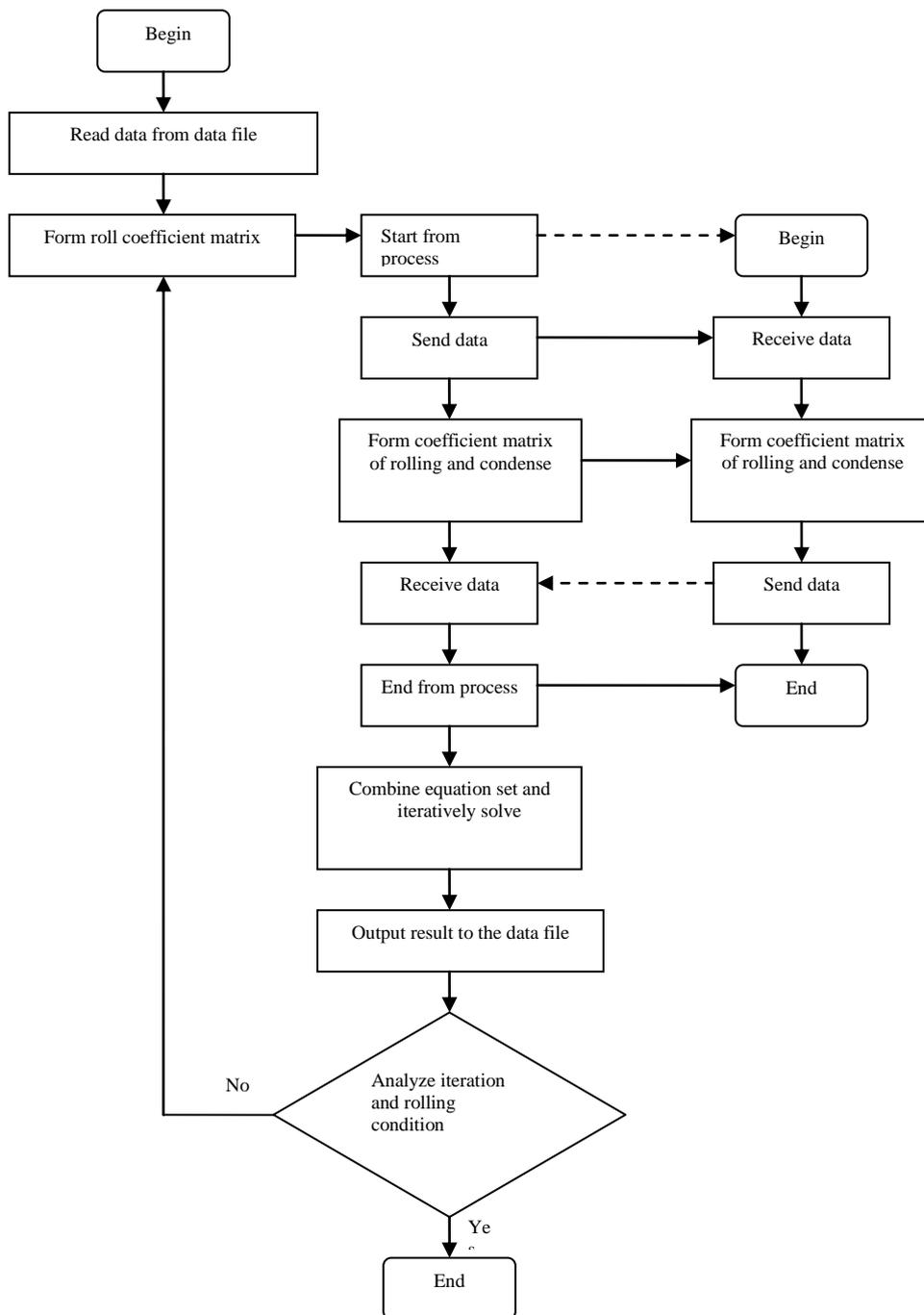


Fig 5. Parallel condensed algorithmic process.

Table 1. Rolling parameter table.

Roll			Strip		
Young modulus	$E / GPa$	210	Young modulus	$E / GPa$	150
Poisson ratio	$\nu$	0.3	Poisson ratio	$\nu$	0.3
Roll radius	$R / mm$	150	Yield stress	$e_s / MPa$	280
Length of roll	$L / mm$	128	Maximum shear stress	$k / MPa$	162
Initial roll gap	$h_1 / mm$	1.35	Thickness	$h_0 / mm$	1.5
Coulomb friction coefficient		0.008	Hardening coefficient	$H$	0.001
Shear friction coefficient	$m$	0.5	Plate width	$b / mm$	120

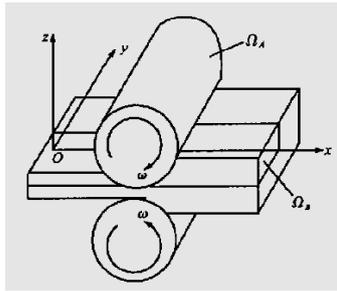


Fig 6. Strip rolling.

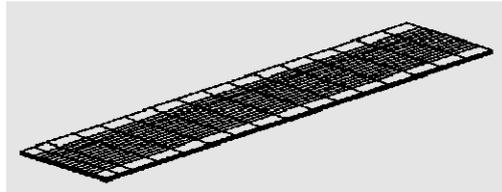


Fig 7. Rolling element.

In two computers connected with Ethernet, we use the parallel algorithm to get the calculation result, see Fig. (8)-(11). The Fig. (8) is rolling pressure distribution between roll and rolling piece and the maximum is 900MPa and the contact area length of strip center is 4.2mm. The Fig. (9) is friction stress distribution on the direction of rolling. The Fig. (10) is friction stress distribution on the direction of plate width. From Fig. (11), due to the elastic deformation of roll, the maximum rolling pressure occurs near the edge.

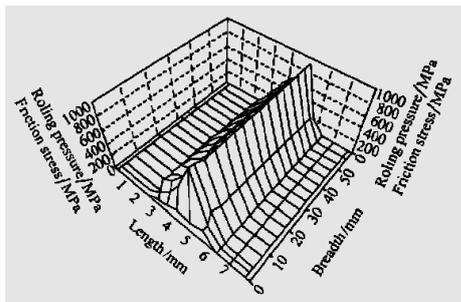


Fig 8. Rolling force distribution.

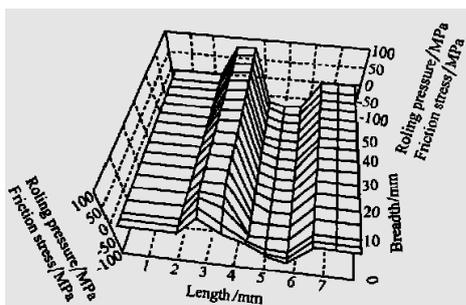


Fig 9. Friction stress distribution along the rolling direction.

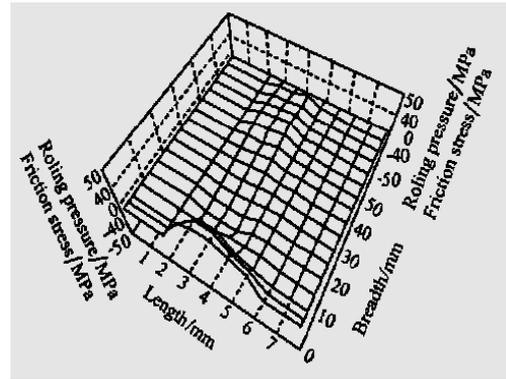


Fig 10. Friction stress distribution on the width direction.

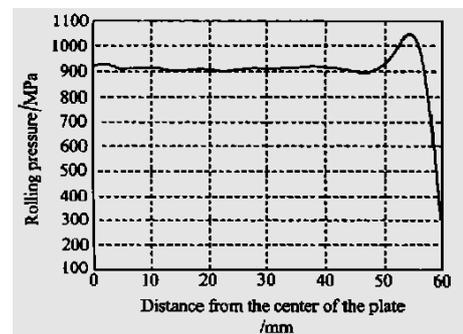


Fig 11. Fringe influence on the rolling stress.

Further, we use adaptive control technology for numerical calculation of rolling force and find out the calculated value of rolling force. Then we compare it with the result of not using adaptive control technology model, see Fig. (12). From comparative analysis we know that when not using adaptive control technology, mean square error of calculated value and measured value is  $\sigma = 14.9\%$ ; when using adaptive control technology, mean square error is  $\sigma = 8.4\%$ . So our algorithm can significantly improve the setting precision of the rolling force and meet the requirements of online control.

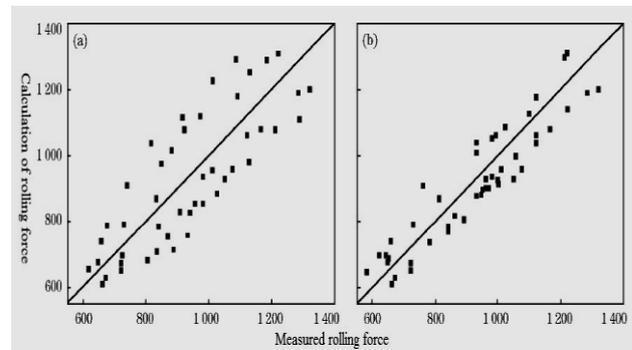


Fig 12. Comparison of rolling force calculated and measured values.

## 4 CONCLUSIONS

In the paper, we combine three-dimensional elastic-plastic contact BEM parallel condensed algorithm with the adaptive algorithm, then numerically simulate the setting of the rolling force. Examples prove that our algorithm obtains high parallel speedup ratio and parallel efficiency and improve the calculation accuracy and computational efficiency.

## 5 ACKNOWLEDGEMENTS

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